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Reg. No.................................

Name.....................................

M.Sc. DEGREE (C.S.S.) EXAMINATION, JANUARY 2017

Third Semester

Faculty of Science

Branch I (A) : Mathematics

MT 03 C15—OPTIMIZATION TECHNIQUES

(2012 Admission onwards)

Time : Three Hours

Maximum Weight : 30

Part A

Answer any **five** *questions.*
Each question has weight 1.

1. What is integer programming ?
2. How does the optimal solution of an integer programming problem compared with that of the linear programming problem ?
3. Define the following :
   - (a) Pure strategies.
   - (b) Mixed strategies.
   - (c) Rectangular game.
4. Define a saddle point. Is it necessary that a game should always possess a saddle point ?
5. Define the following :
   - (a) Tree.
   - (b) Centre of a graph.
   - (c) Arborescence.
6. What do you mean by sensitivity analysis ?
7. Define the following terms :
   - (a) Convex set.
   - (b) Hessian matrix.
8. What is a ray solution and what is its significance in complementary Pivot method ?

$(5 \times 1 = 5)$

Turn over

Part B

*Answer any **five** questions.*
Each question has weight 2.

9. Explain how Gomory's cutting plane algorithm works.

10. Solve using cutting plane method

Minimise $4x_1 + 5x_2$
subject to $3x_1 + x_2 \geq 2$
$x_1 + 4x_2 \geq 5$
$3x_1 + 2x_2 \geq 7$
x_1, x_2 non-negative integers.

11. Find the minimum spanning tree in the following undirected graph :

Arc :	(1, 2)	(1, 3)	(1, 4)	(2, 3)	(2, 8)	(2, 10)	(3, 4)	(3, 8)	(4, 5)	(4, 6)
Length :	7	4	8	3	9	14	4	10	15	12
Arc :	(4, 8)	(5, 6)	(5, 7)	(6, 7)	(6, 8)	(6, 9)	(7, 9)	(8, 9)	(8, 10)	(9, 10)
Length :	10	4	1	2	20	16	18	3	4	6

12. Discuss the changes in the co-efficients a_{ij} for the given L_p problem :

Maximum $Z = cx$, subject to $Ax = b$, $X \leq 0$.

13. Solve graphically the game whose pay-off matrix is given by :

$$\begin{bmatrix} 2 & 7 \\ 3 & 5 \\ 11 & 2 \end{bmatrix}.$$

14. Describe the maximin principle of game theory.

15. Use Newton's method to solve the problem :

Minimize $y(x) = 2(x_1 + x_2)^2 + 2(x^2_1 + x^2_2)$.

16. Solve using constrained derivatives :

Minimize $P(x) = 7x_1 - 6x_2 + 4x_3$
subject to $x_1^2 + 2x_2^2 + 3x_3^2 = 1$
$x_1 + 5x_2 - 3x_3 = 6$.

(5 × 2 = 10)

Part C

*Answer any **three** questions.*
Each question has weight 5.

17. In a factory 4000 units of a certain product are to be manufactured. There are three machines on which it can be manufactured. The set up cost, the production cost per unit and the maximum production capacity for each machine are tabulated below. The objective is to minimize the total cost of producing the entire lot. Formulate the problem as an integer programming and solve it :

Machine		Set up cost	Production cost per unit	Capacity units
I	...	400	10	2,400
II	...	600	4	1,600
III	...	200	20	1,200

18. State and prove minimax theorem.

19. A factory can manufacture two products A and B. The profit on a unit of A is Rs. 80 and of B Rs. 40. The maximum demand of A is 6 units per week and of B is 8 units. The manufacturer has set up a goal of achieving a profit of Rs. 640 per week. Formulate the problem as a goal programming and solve it.

20. Use the notion of dominance to simplify the following pay-off matrix and then solve the game :

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 4 & 2 & 0 & 2 & 1 \\ 4 & 3 & 1 & 3 & 2 \\ 4 & 3 & 4 & -1 & 2 \end{bmatrix}.$$

21. Describe the complementary Pivot method to solve complementary problems.

22. (a) Use the method of Lagrange multipliers to solve the following problem. Does the solution maximize or minimize the objective function :

$$\text{Optimize } Z = x_1^2 - 10x_1 + x_2^2 - 6x_2 + x_3^2 - 4x_3$$
$$\text{subject to } x_1 + x_2 + x_3 = 7 \text{ and}$$
$$x_1, x_2, x_3 \geq 0.$$

(b) Write the Kuhn-Tucker conditions for the problem :

$$\text{Minimize } Z = x_1^2 + x_2^2 + x_3^2$$
$$\text{subject to } 2x_1 + x_2 - x_3 \leq 0$$
$$1 - x_1 \leq 0$$
$$2 - x_2 \leq 0$$
$$-x_3 \leq 0.$$

Also solve the problem.

(3 × 5 = 15)