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Reg. No.................................

Name.....................................

M.Sc. DEGREE (C.S.S.) EXAMINATION, JANUARY 2016

Third Semester

Faculty of Science

Branch I (a)—Mathematics

MT 03C 11—MULTIVARIATE CALCULUS AND INTEGRAL TRANSFORMS

(2012 Admission onwards)

Time : Three Hours | Maximum Weight : 30

Part A

Answer any **five** *questions.*
Each question has weight 1.

1. Write the Fourier series generated by a function *f* with period 2π in terms of complex exponentials.
2. Define integral transform of a function *f*. Write the exponential Fourier transform.
3. Show that the total derivative of a linear function is the function itself.
4. Define the Jacobian matrix of a function $f : \mathrm{R}^n \to \mathrm{R}^m$.
5. State inverse function theorem.
6. If $f = u + iv$ is a complex valued function with s derivative at a point $Z \in C$, show that :

   $J_f(z) = |f'(z)|^2$.
7. Define a flip.
8. Define support of a function *f* on R^k.

$(5 \times 1 = 5)$

Part B

Answer any **five** *questions.*
Each question has weight 2.

9. Use Fourier integral theorem to evaluate $\int_0^\infty \frac{x \sin \alpha x}{1 + x^2}, \alpha \neq 0$.
10. State and prove Weirstrass appoximation theorem.
11. State and prove the sufficient conditions for differentiability of $f = u + iv$.

Turn over

12. Compute the gradient vector $\nabla f(x,y)$ at those points (x, y) in R^2 where if exists for the function :

$f(x,y) = x^2y^2\log(x^2+y^2)$ if $(x,y) \neq (0,0)$, $f(0,0) = 0$.

13. Let f be the complex valued function defined by $f(z) = 1/\bar{z}, z \neq 0$, show that :

$J_f(z) = -|z|^{-4}$.

14. Let S be an open connected subset of R^n and let $f : S \to R^m$ be differentiable at each point of S. If $f'(c) = 0$ for each c in S, show that f is constant on S.

15. If w is of class $\mathscr{C}''$ in E, then show that $d^2w = 0$.

16. Suppose E is an open set in R^n, and T is a $\mathscr{C}'$ - mapping of E into an open set $V \subset R^m$ and ω is a k-form in V. Show that $d(\omega_T) = (d\omega)_T$ if ω is of class $\mathscr{C}'$ and T is of class $\mathscr{C}''$.

(5 × 2 = 10)

Part C

Answer any **three** *questions.*
Each question has weight 5.

17. State and prove Fourier-integral theorem.

18. State and prove chain rule for total derivatives.

19. (a) Let $f : R^2 \to R^3$ be defined by $f(x,y) = (\sin x\cos y, \sin x\sin y, \cos x\cos y)$. Determine the Jacobian matrix $Df(x,y)$.

(b) Let $f : S \subset R^n$ to R^m, show that if f is differentiable at c, then f is continuous at c.

20. Let $f : R^n \to R^m$ and if both partial derivative $D_r f$ and $D_k f$ exist in an n-ball B (c, δ) and if both are differentiable at c, then prove that $D_{r,k}f(c) = D_{k,r}f(c)$.

21. Let $f = (f_1, f_2, \ldots, f_n)$ has continuous partial derivatives $D_j f_i$ on an open set S in R^n and that the Jacobian $J_f(a) \neq 0$ for some point a in S. Show that there is an n-ball B (a) on which f is one-one.

22. For $(x,y) \in R^2$ define $F(x,y) = (e^x\cos y - 1, e^x\sin y)$. Prove that $F = G_2 O G_1$, where :

$G_1(x,y) = (e^x\cos y - 1, y)$ $G_2(u,v) = (u, (1+u)\tan v)$ are primitive in some neighborhood of (0, 0).

(3 × 5 = 15)