F 4673 (Pages : 3) Reg. No.................................

Name.................................

M.Sc. DEGREE (C.S.S.) EXAMINATION, JANUARY 2017

Third Semester

Faculty of Science

Branch I (A) : Mathematics

MT 03 C11—MULTIVARIATE CALCULUS AND INTEGRAL TRANSFORMS

(2012 Admission onwards)

Time : Three Hours Maximum Weight : 30

Part A

Answer any **five** *questions.*
Each question has weight of 1.

1. Define exponential transform, Fourier sine transform and Fourier cosine transforms.

2. Define convolution of two functions.

3. Give an example of a function that can have a finite directional derivative $f'(c, u)$ for every u but may fail to be continuous at c.

4. Write the Jacobian matrix of a $f : \mathrm{R}^n \to \mathrm{R}^m$ which is differentiable at c.

5. State implicit function theorem.

6. Give an example of a function $f(x, y)$ where $D_{1,2}\ f(x, y) \neq D_{2,1}\ f(x, y)$.

7. State Stokes theorem.

8. Define a flip. Give an example.

$(5 \times 1 = 5)$

Part B

Answer any **five** *questions.*
Each question has weight of 2.

9. Let $\mathrm{R} = (-\infty, \infty)$. Assume that $f \in \mathrm{L}(\mathrm{R})$, $g \in \mathrm{L}(\mathrm{R})$ and that either f or g is bounded on R. Show that the integral $h(x) = \int_{-\infty}^{\infty} f(t)\, g(x-t)\, dt$ exists for every $x \in \mathrm{R}$ and that h is bounded on R.

Turn over

10. State and prove the exponential form of Fourier integral theorem.

11. Prove that if f is differentiable at c then f is continuous at c.

12. Let $f : \mathbb{R}^2 \to \mathbb{R}^3$ be defined by $f(x, y) = (\sin x \cos y, \sin x \sin y, \cos x \cos y)$. Determine the Jacobian matrix $\mathrm{D} f(x, y)$.

13. State and prove Mean Value theorem for vector valued functions.

14. Let A be an open subset of $\mathbb{R}^n$ and assume that $f : \mathrm{A} \to \mathbb{R}^n$ is continuous and has finite partial derivatives $\mathrm{D}_j f_i$ on A. Prove that if f is one-to-one on A and if $\mathrm{J}_f(x) \neq 0$ for each $x \in \mathrm{A}$, then $f(\mathrm{A})$ is open.

15. If ω is of class ζ^n in E, show that $d^2\omega = 0$.

16. For every $f \in \zeta(\mathrm{I}^k)$, show that $\mathrm{L}(f) = \mathrm{L}'(f)$.

$(5 \times 2 = 10)$

Part C

Answer any **three** *questions.*
Each question has weight of 5.

17. State and prove Fourier integral theorem.

18. Assume that g is differentiable at a, with total derivative $g'(a)$. Let $b = g(a)$ and assume that f is differentiable at b, with total derivative $f'(b)$. Prove that the composition $h = f \circ g$ is differentiable at a and the total derivative $h'(a)$ is given by $h'(a) = f'(b) \circ g'(a)$.

19. (a) Compute the gradient $\nabla f(x, y)$ in $\mathbb{R}^2$ for $f(x, y) = x^2 y^2 \log(x^2 + y^2)$ if $(x, y) \neq (0, 0)$, $f(0, 0) = 1$.

    (b) Let u and v be two real-valued functions defined on a subset S of the complex plane. Assume also that u and v are differentiable at an interior point c of S and that the partial derivatives satisfy C.R. equations at c. Prove that $f = u + i\mathrm{V}$ has a derivative at c and $f'(c) = \mathrm{D}_1 u(c) + i\,\mathrm{D}_1 v(c)$.

20. State and prove the sufficient condition for the equality of mixed partial derivatives.

21. State and prove the second derivative test for extrema.

22. Suppose k is a compact subset of R^4 and $\{V_\alpha\}$ is an open cover of k. Show that there exists functions

$\psi_1, \psi_2 \ldots\ldots, \psi_s \in \zeta\left(R^4\right)$ such that :

(a) $0 \le \psi_i \le 1$ for $1 \le i \le s$.

(b) Each ψ_i has its support in some V_{α}.

(c) $\psi_1(x) + \psi_1(x) + \ldots\ldots + \psi_s(x) = 1$ for every $x \in k$.

(3 × 5 = 15)