QP CODE: 21000384

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Reg No : ....

Name : ....

M Sc DEGREE (CSS) EXAMINATION, MARCH 2021

Third Semester

Faculty of Science

CORE - ME010304 - FUNCTIONAL ANALYSIS

M Sc MATHEMATICS,M Sc MATHEMATICS (SF)

2019 Admission Onwards

842D9303

Time: 3 Hours

Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. Define a Cauchy sequence. Give an example.
2. Prove that an infinite dimensional subspace of a normed space need not be closed.
3. Prove that $\|T_1T_2\| \leq \|T_1\|\|T_2\|$ and $\|T^n\| \leq \|T\|^n$
4. Let $f : C[a,b] \to R$ be a linear function defined by $f(x) = x(t_0), t_0 \in [a,b]$ Is f a bounded functional on $C[a,b]$? Justify
5. Let X be a finite dimensional vector space. If $x_0 \in X$ has the property that $f(x_0) = 0$ for all $f \in X^*$, then $x_0 = 0$
6. Define projection operator. Show that it is idempotent.
7. Define an orthonormal set. Show that it is linearly independent.
8. Prove that if, $\langle v_1, w\rangle = \langle v_2, w\rangle$ for all w in an innerproduct space X then $v_1 = v_2$
9. *Define totally ordered set. Give an example.*
10. *Let X, Y and Z are normed spaces and $T \in B(X,Y)$ and $S \in B(Y,Z)$ then prove that $(ST)^\times = T^\times S^\times$.*

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

Weight 2 each.

11. Prove that a metric d induced by a norm satisfies translation invariance.

12. Prove that a closed and bounded set in a metric space need not be compact.

13. Define Null space and Range of a linear operator and prove that they are vector spaces.

14. Prove that in a finite dimensional normed space X, every linear operator is bounded.

15. Show that the space l^p with $p \neq 2$ is not an inner product space,

16. Prove that for any x in an inner product space X can have atmost countably many nonzero Fourier coefficients $\langle x, e_k \rangle$ with respect to an orthonormal family $(e_k), k \in I$ in X.

17. *Define Hilbert-adjoint operator. Let* H_1 *and* H_2 *are Hilbert spaces and* $S, T \in B(H_1, H_2)$ *then prove that* $(S+T)^* = S^* + T^*$ and $(\alpha T)^* = \bar{\alpha} T^*$ *where* α *be any scalar.*

18. *Prove that the product of two bounded linear operators on a Hilbert space is self-adjoint if and only if the operators commute.*

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. (i) Define complete metric space.
    (ii) Show that $l^p, 1 \leq p < \infty$ is complete.
    (iii) Show that Q, the set of rational numbers with respect to usual metric is incomplete.

20. Show that dual space of l^p is l^q $1 < p < \infty$, $\frac{1}{p} + \frac{1}{q} = 1$

21. Prove that an Orthonormal set M in a Hilbert space H is total in H if and only if for all $x \in H$ the parseval relation holds.

22. *State and prove Hahn-Banach theorem of extension of linear functionals.*

(2×5=10 weightage)