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Reg. No.................................

Name....................................

M.Sc. DEGREE (C.S.S.) EXAMINATION, JUNE 2018

Second Semester

Faculty of Science

Branch I (A) : Mathematics

MT 02 C09—PARTIAL DIFFERENTIAL EQUATIONS

(2012 Admission onwards)

Time : Three Hours — Maximum Weight : 30

Part A

Answer any **five** *questions.*
Each question carries weight 1.

1. Solve : $y\,dx + x\,dy + 2z\,dz = 0$.

2. Eliminate a and b from $z = x + ax^2y^2 + b$.

3. Find the complete integral : $zpq - p - q = 0$.

4. Define compatible system of first order equations.

5. Prove $F\left(D, D^1\right) e^{ax+by} = F(a, b)\, e^{ax+by}$.

6. Eliminate the arbitrary functions :

$$z = f\left(x^2 - y\right) + g\left(x^2 + y\right).$$

7. Explain : interior Dirichlet problem.

8. Show that $\psi = \dfrac{q}{\left|\vec{r} - \vec{r}^1\right|}$ is a solution of $\nabla^2 \psi = 0$.

(5 × 1 = 5)

Part B

Answer any **five** *questions.*
Each question carries weight 2.

9. Find the general integral : $z(xp - yq) = y^2 - x^2$.

Turn over

10. Eliminate the constants a and b from :

$$z^2\left(1+a^3\right)=8\left(x+ay+b\right)^3.$$

11. Show that $xp-yq=x$ and $x^2\ p+q=xz$ are compatible and find their solutions.

12. Find the complete integral of the separable type of equation $g\left(x,\ p\right)=h\left(y,\ q\right)$.

13. Reduce to canonical form :

$$u_{xx}+x^2\ u_{yy}=0.$$

14. If the operator $\mathrm{F}\left(\mathrm{D},\mathrm{D}^1\right)$ is reducible, show that the order in which the linear factors occur is unimportant.

15. Solve $z\left(qs-pt\right)=pq^2$.

16. Show that the surfaces :

$$x^2+y^2+z^2=c\,x^{2/3}$$

can form a family of equipotential surfaces and find the general form of the corresponding potential function.

(5 × 2 = 10)

Part C

Answer any **three** *questions.*
Each question carries weight 5.

17. Prove : A necessary and sufficient condition that the Pfaffian differential equation $\overline{\mathrm{X}}\cdot d\bar{r}=0$ should be integrable is that $\overline{\mathrm{X}}\cdot\mathrm{curl}\,\overline{\mathrm{X}}=0$.

18. Find the integral surface of :

$$x^3p+y\left(3x^2+y\right)q=z\left(2x^2+y\right)$$

which passes through the curve

$$x_0=1\ y_0=s,\ z_0=s\left(1+s\right).$$

19. By Jacobi's method solve :

$$z^2 + zu_z - u_x^2 - u_y^2 = 0.$$

20. Solve by Cauchy's method of characteristics :

$$z = \frac{1}{2}\left(p^2 + q^2\right) + (p - x)(q - y)$$

Which passes through the x-axis.

21. Show that $z = f(u) + g(v) + w$ is a solution of the second order linear partial equation :

$$\mathrm{R}r + \mathrm{S}s + \mathrm{T}t + \mathrm{P}p + \mathrm{Q}q = w$$

where R, S, T, P, Q, W are known functions of x and y.

22. Find the complete integral and general integral of :

$$r + 4s + t + rt - s^2 = 2.$$

(3 × 5 = 15)