G 18001482

18001482

Reg. No.................................

Name....................................

M.Sc. DEGREE (C.S.S.) EXAMINATION, JUNE 2018

Second Semester

Faculty of Science

Branch I (a) : Mathematics

MT0 2C 06—ABSTRACT ALGEBRA

(2012 Admission onwards)

Time : Three Hours | Maximum Weight : 30

Part A

Answer any **five** *questions.*
Each question carries a weight of 1.

1. State the fundamental theorem of finitely generated abelian groups.

2. Show that $f(x) = x^3 + 3x + 2$ viewed in $Z_5[x]$ is irreducible over Z_5.

3. Show that $\sqrt{1+\sqrt{3}}$ is algebraic over Q.

4. Define : Galois field of order p^n.

5. Prove : The center of a non-trivial p-group G is non-trivial.

6. Show that the set of all automorphisms of a field E is a group under function composition.

7. Give examples for normal subgroup and factor group.

8. Prove : Every field of characteristics zero is perfect.

(5 × 1 = 5)

Part B

Answer any **five** *questions.*
Each question carries a weight of 2.

9. Obtain necessary and sufficient conditions for the group $Z_m \times Z_n$ to be isomorphic to Z_{mn}.

Turn over

10. If m divide the order of a finite abelian group G, prove that G has a subgroup of order m.

11. Explain :

    (i) Tower of fields.

    (ii) Algebraic number.

    (iii) Simple extension of a field.

    (iv) Irreducible polynomial.

12. If F is a field of prime characteristic p, then prove that $(\alpha + \beta)^{p^n} = \alpha^{p^n} + \beta^{p^n}$ for all $\alpha, \beta \in F$ and all positive integers n.

13. Use Sylow theorems to show that no group of order 15 is simple.

14. Establish the theorem exhibiting the Frobenius automorphism.

15. Show by an example that the phenomenon of a zero of multiplicity greater than 1 of an irreducible polynomial can occur.

16. Define extension field of a field and a splitting field with examples. Also explain your examples.

(5 × 2 = 10)

Part C

Answer any **three** *questions.*
Each question carries a weight of 5.

17. Establish Eisenstein criteria for irreducibility. Also show that the cyclotomic polynomial $x^{p-1} + x^{p-2} + ... + x + 1$ is irreducible over Q for any prime p.

18. State and prove the evaluation homomorphism theorem for field theory.

19. Establish "Trisecting the angle is impossible". State the theorem and its corollary that are used in this proof.

20. Prove :

    (i) A finite extension E of a finite field F is a simple extension of F.

    (ii) A finite field E of p^n elements is the splitting field of $x^{p^n} - x$ over its prime subfield Z_p.

21. Define :

(i) Subfield.

(ii) Finite field.

(iii) Automorphism of fields.

(iv) Group of E over F.

(v) Algebraic extension of a field.

(vi) Conjugate elements of algebraic extension of a field.

(vii) Fixed field.

(viii) Permutation of a field.

(ix) Algebraic closure of a field.

(x) Finite extension of a field.

22. (i) If K is an finite extension of E and E is a finite extension of F show that K is separable over F if and only if K is separable over E and E is separable over F.

(ii) If E is a finite extension of F prove that E is separable over F if and only if each α in E is separable over F.

$(3 \times 5 = 15)$