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Reg. No.................................

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M.Sc. DEGREE (C.S.S.) EXAMINATION, MAY 2017

Fourth Semester

Faculty of Science

Branch I (A) : Mathematics

MT 04 C16—SPECTRAL THEORY

(Programme – Core – Common for all)

[2012 Admissions – Regular]

Time : Three Hours

Maximum Weight : 30

Part A

Answer any **five** *questions.*
Each question has weight 1.

1. Define weak convergence in a normed space. Show that the strong convergence implies weak convergence with the same limit.
2. Show that a contraction T on a metric space X is a continuous mapping.
3. Show that the eigen values of a skew-Hermitian matrix $A = (\alpha_{jk})$ are pure imaginary or zero.
4. Define Banach algebra. Give an example.
5. Prove that $T : l^2 \to l^2$ defined by $T(\xi_j) = (\xi_j / 2^j)$ for $j = 1, 2, \ldots$ is compact.
6. Let X and Y be normed spaces. Show that $T : X \to Y$ is compact if and only if it maps every bounded sequence (x_n) in X onto a sequence $T(x_n)$ in Y which has a convergent subsequence.
7. Let S and T be bounded self-adjoint linear operators on a complex Hilbert space. If $S \leq T$ and $S \geq T$ then, show that $S = T$.
8. If P_n is a sequence of projections on Hilbert space H and $P_n \to P$, show that P is projection on H.

(5 × 1 = 5)

Part B

Answer any **five** *questions.*
Each question has weight 2.

9. Show that T^n $(n \in \mathbb{N})$ is a contraction, if T is a contraction.
10. If $x_n \in C[a, b]$ and $x_n \xrightarrow{w} x \in C[a, b]$ show that (x_n) is pointwise convergent on $[a, b]$.

Turn over

11. State and prove closed graph theorem.

12. Prove that all matrices representing a given linear operator $T : X \to X$ on a finite dimensional normed space X relative to various bases for X have the same eigen values.

13. Show that the spectrum of a bounded linear operator on a complex Banach space is closed.

14. Find a linear operator $T : C[0,1] \to C[0,1]$ whose spectrum is a given interval $[a, b]$.

15. Let X, Y be normed spaces. Show that the range $\mathscr{R}(T)$ of a compact linear operator $T : X \to Y$ is separable.

16. Show that the spectrum $\sigma(T)$ of a bounded self adjoint linear operator $T : H \to H$ on a complex Hilbert space is real.

$(5 \times 2 = 10)$

Part C

Answer any **three** *questions.*
Each question has weight 5.

17. Let A be a complex Banach algebra with identity e. For any $x \in A$, show that the spectrum $\sigma(x)$ is non-empty compact subset and the spectral radius $r_\sigma(x) \leq \|x\|$.

18. State and prove open mapping theorem.

19. If (x_n) and (y_n) are Cauchy sequences in a normed algebra A, show that :

    (a) $(x_n y_n)$ is Cauchy in A.

    (b) If $x_n \to x$ and $y_n \to y$ then, $x_n y_n \to xy$.

20. Suppose that T is densely defined injective operator in a complex Hilbert space with $\mathscr{R}(T)$ is dense. Prove that T^* is injective and $(T^*)^{-1} = (T^{-1})^*$.

21. If two bounded self-adjoint linear operator S and T on a Hilbert space H are positive and commutes, then show that ST is positive.

22. Prove that a bounded linear operator $P : H \to H$ on a Hilbert space H is a projection if and only if P is self adjoint and idempotent.

$(3 \times 5 = 15)$