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Reg. No.................................

Name....................................

M.Sc. DEGREE (C.S.S.) EXAMINATION, MAY 2017

Fourth Semester

Faculty of Science

Branch I (A)—Mathematics

MT 04E 14—CODING THEORY

(2012 Admissions—Regular)

Time : Three Hours

Maximum Weight : 30

Part A

Answer any **five** *questions.*
Each question has weight 1.

1. Describe Maximum-likelyhood decoding.
2. Define perfect code. Give an example.
3. List all binary, irreducible polynomials of degree less than or equal to 5.
4. Define self-dual and self-orthogonal codes.
5. Prove that $(x \pm y)^{p^n} = x^{p^n} \pm y^{p^n}$ for all x, y in a field F of characteristic p.
6. Show that minimal polynomial $m(x)$ of an element α in a finite field $GF(p^r)$ is irreducible.
7. Obtain a generating polynomial of a single error-correcting ternary BCH code of length 8.
8. Define cyclic code. Give an example.

$(5 \times 1 = 5)$

Part B

Answer any **five** *questions.*
Each question has weight 2.

9. Show that in a binary code either all the vectors have even weight or half have even and half have odd weights.
10. Show that $A(n-1, d-1) = A(n, d)$ if d is even.

Turn over

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11. Construct a finite field of 16 elements.

12. Explain Double error-correcting BCH code.

13. Prove that every finite field has a primitive element.

14. Find a generator matrix of a $[5, 3, 3]$ MDS code over GF (4).

15. If $e(x)$ is an idempotent, then show that $e(x)$ is orthogonal to $\left(1-e\left(x^{-1}\right)\right)$.

16. Describe Reed-Solomon code. Prove that it is an MDS code.

$(5 \times 2 = 10)$

Part C

Answer any **three** *questions.*
Each question has weight 5.

17. (a) If the rows of a generator matrix G for a $[n, k]$ code C have weights divisible by 4 and are orthogonal to each other, then prove that C is self orthogonal and all weights in C are divisible by 4.

(b) Give a parity check matrix for the $[7, 4]$ - Hamming code.

18. (a) Prove that a binary code of length n, minimum distance d or more and dimension $k \geq n-m$ exists if :

$$\binom{n-1}{1}+\binom{n-1}{2}+\ldots\ldots+\binom{n-1}{d-2}<2^m-1.$$

(b) Prove that any binary $[23, 12, 7]$ code is perfect.

19. (a) Show that minimal polynomial $m(x)$ of an element α in a finite field GF $\left(p^r\right)$ divides $x^{p^r}-x$

(b) How many polynomials of the form x^2+ax+b with $b \neq 0$ are there over GF (4).

20. (a) Define the binary [24,12] Golay code. Show that its minimum weight is 8 and corrects triple errors.

    (b) Find a quadratic polynomial which is irreducible over GF (16).

21. (a) Find all binary cyclic codes in R_5.

    (b) Explain the method of finding cyclic codes.

22. (a) Explain the method of decoding BCH codes.

    (b) Find a generator polynomial for a double-error-correcting Reed-Solomon code over GF (16). Give its length and dimension.

$(3 \times 5 = 15)$