G 2352 (Pages : 2) Reg. No.................................

Name...................................

M.Sc. DEGREE (C.S.S.) EXAMINATION, JUNE 2015

Fourth Semester

Faculty of Science

Branch I (A)—Mathematics

MT 04 E01—ANALYTIC NUMBER THEORY

(2012 Admission onwards)

[Regular/Supplementary]

Time : Three Hours Maximum Weight : 30

Part A

Answer any **five** *questions.*
Each question has weight 1.

1. Prove that $d(n)$ is odd if and only if n is a square.

2. Prove that every number of the form $2^{a-1}(2^a - 1)$ is perfect if $2^a - 1$ is prime.

3. What are the possible values of $\{x\}$ and $\{-x\}$ where $\{x\} = x - [x]$ is the fractional part of x.

4. Prove that for every $n > 1$ there exist n consecutive composite numbers.

5. Prove or disprove : Every prime $p75$ can be expressed in the form $30\,m + n$ where $m \geq 0$ and $n \in \{1, 7, 11, 13, 17, 19, 23, 29\}$.

6. Find all x which simultaneously satisfy the system of congruences.
$x \equiv 1(\text{mod}\,3)$, $x \equiv 2\,\text{mod}\,4$, $x \equiv 3\,\text{mod}\,5$.

7. Let a, b, x_0 be positive integers. Define $x_n = ax_{n-1} + b$ for $n = 1, 2, \ldots$ Prove that not all of x_n can be prime.

8. Obtain the recurrence formula :

$$np(n) = \sum_{k=1}^{n} \sigma(k)\, p(n-k).$$

$(5 \times 1 = 5)$

Turn over

Part B

Answer any **five** *questions.*
Each question has weight 2.

9. Obtain the product form of the Mö bius inversion formula.

10. If f is multiplicative, prove that :

    $f^{-1}(p^2) = f(p^2) - f(p^2)$ for every prime p.

11. Find the density of the set of lattice points visible from the origin.

12. State and prove the theorem connecting the prime number theorem and the n^{th} prime.

13. Find all positive integers n for which $n^{13} \equiv n(\text{mod} 1365)$.

14. If p is an odd prime, let $q = (p-1)/2$. Prove that $(q!)^2 + (-1)^q = 0 \bmod p$.

15. State and prove Wilson's theorem.

16. Prove that 2 is a primitive root mod p if p is a prime of the form $4q + 1$ where q is an odd prime.

(5 × 2 = 10)

Part C

Answer any **three** *questions.*
Each question has weight 5.

17. (a) Define Mö bius function and Euler totient function. Find the relation between them.

    (b) Differentiate between multiplicative function and completely multiplication function with examples.

18. (a) State and prove Euler's summation formula.

    (b) Obtain Mö bius inversion formula.

19. State two equivalent forms of prime number theorem and prove.

20. (a) Show that congruence is an equivalence relation.

    (b) State and prove Chinese remainder theorem.

21. (a) Establish the cancellation law on congruences.

    (b) State and prove Little Fermat Theorem.

22. Establish Euler's pentagonal – number theorem.

(3 × 5 = 15)