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Reg. No...............................

Name...................................

M.Sc. DEGREE (C.S.S.) EXAMINATION, MAY 2017

Fourth Semester

Faculty of Science

Branch I (A)—Mathematics

MT 04E 01—ANALYTIC NUMBER THEORY

(2012 Admissions—Regular)

Time : Three Hours Maximum Weight : 30

Part A

Answer any **five** *questions.*
Each question has weight 1.

1. Define Mobius function. Prove that $\sum_{d^2|n} \mu(d) = \mu^2(n)$.
2. Define Dirichlet product of two arithmetical functions. Show that it is commutative and associative.
3. Show that $\log [x]! = x \log x - x + O(\log(x))$ for $x \geq 2$.
4. State prime number theorem.
5. Calculate the highest power of 10 that divides 1000 !.
6. Show that a common factor which is relatively prime to the modulus can be cancelled in a congruence relation.
7. State Chinese Remainder theorem.
8. Show that there are no primitive roots mod 2^{α}, if $\alpha \geq 3$.

$(5 \times 1 = 5)$

Part B

Answer any **five** *questions.*
Each question has weight 2.

9. For $n \geq 1$, prove that $\phi(n) = n \prod_{p|n} \left(1 - \frac{1}{p}\right)$.
10. State and prove Euler's summation formula.
11. For 'p' prime, $x \geq 2$ prove that : $\sum_{p \leq x} \left[\frac{x}{p}\right] \log p = x \log x + O(x)$.

Turn over

12. State and prove Abel's identity.

13. State and prove Langrange's theorem.

14. State and prove Wolstenhome's theorem.

15. Assume $(a, m) = d$ and suppose $d|b$, show that the linear congruence $ax \equiv b \pmod{m}$ has exactly 'd' solutions.

16. Write a short note on Euler's pentagonal-number theorem.

$(5 \times 2 = 10)$

Part C

Answer any **three** *questions.*
Each question has weight 5.

17. Prove that the set of lattice points visible from the origin has density $\frac{6}{\pi^2}$.

18. (a) Define generalized convolution. For any arithmetical functions α and β, show that

$$\alpha \circ (\beta \circ F) = (\alpha * \beta) \circ F.$$

(b) Show that the average order of $\phi(n)$ is $\frac{3n}{\pi^2}$.

19. State and prove Shapiro's Tauberian theorem.

20. State and prove Chinese Remainder theorem. Find all x which simultaneously satisfy the system of congruences

$x \equiv 1 \pmod{3}$, $x \equiv 2 \pmod{4}$, $x \equiv \pmod{5}$.

21. Prove that the set of lattice points in the plane visible from the origin contains arbitrarily large gaps.

22. For $|x| < 1$ prove that $\prod_{m=1}^{\infty} \frac{1}{1-x^m} = \sum_{n=0}^{\infty} p(n) x^n$, where $p(0) = 1$.

$(3 \times 5 = 15)$