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Reg. No................................

Name...................................

M.Sc. DEGREE (C.S.S.) EXAMINATION, MAY 2018

Fourth Semester

Faculty of Science

Branch I (A) : Mathematics

MT 04 E02—COMBINATORICS

(2012 Admission onwards)

Time : Three Hours | Maximum Weight : 30

Part A

Answer any **five** *questions.*
Each question has weight 1.

1. Find the number of ordered pairs (x, y) of integers such that $x^2 + y^2 \leq 5$.

2. (i) State the injection and bijection principles.

   (ii) Find the number of ternary sequences of length 10 having two 0's, three 1's and five 2's.

3. Show that among any group of 7 people, there must be atleast 4 of the same sex.

4. State generalised Pigeonhole principle.

5. How many arrangements of $a, a, a, b, b, b, c, c, c$ are there such that no three consecutive letters are the same ?

6. Find the number of integers in the set $\left\{10^3, 10^3 + 1, ... 10^n\right\}$ which are not of the from n^2 or $n^3, n \in \mathrm{N}$.

7. Express the generating function for the sequence (C_r) in closed form where $\mathrm{C}_r = r^2$ for each $r \in \mathrm{N}$.

8. In how many ways can 4 of the letters from PAPAYA be arranged.

(5 × 1 = 5)

Turn over

Part B

Answer any **five** *questions.*
Each question has weight 2.

9. There are 7 boys and 3 girls in a gathering. In how many ways can they be arranged in a row so that (i) the 3 girls form a single block ; (ii) the two end positions are occupied by boys and no girls are adjacent.

10. Let $X = \{1, 2, \ldots\ldots n\}$ where $n \in N$. Show that the number of r-combinations of X which contain no consecutive integers is given by $\binom{n-r+1}{r}$ where $0 \le r \le n-r+1$.

11. Show that for any set of 10 points chosen within a square whose sides are of length 3 units, there are two points in the set whose distance apart is at most $\sqrt{2}$.

12. Prove that at a gathering of any six people some three of them are either mutual acquaintances or complete strangers to one another.

13. Let $S = \{1, 2, \ldots\ldots 500\}$. Find the number of integers in S which are divisible by 2, 3 or 5.

14. Find the number of non-negative integer solutions to the equation $x_1 + x_2 + x_3 = 15$ where $x_1 \le 5$, $x_2 \le 6$ and $x_3 \le 7$.

15. Let $k, n \in N$ with $k \le n$. Prove that the number of partitions of n into k parts is equal to the number of partitions of n into parts the largest size of which is k.

16. Solve the recurrence relation $a_n = a_{n-1} + a_{n-2}$ given that $a_0 = 1, a_1 = 1$.

(5 × 2 = 10)

Part C

Answer any **three** *questions.*
Each question has weight 5.

17. (a) Show that $s(r, n) = s(r-1, n-1) + (r-1)\, s(r-1, n)$ where $r, n \in N$ with $n \le r$.

(b) If $|x| = n$ show that $|P(x)| = 2^n$ for all $n \in N$.

18. (a) Seventeen people correspond by mail with one another-each one with all the rest. In their letters only 3 different topics are discussed. Each pair of correspondents deals with only one of these topics. Prove that there are atleast 3 people who write to one another about the same topic.

    (b) For all integers $p, q \geq 2$, show that $R(p, q) \leq R(p-1, q) + R(p, q-1)$.

19. A permutation of $x_1, x_2, \ldots\ldots x_{2n}$ of the set $\{1, 2, \ldots\ldots 2n\}$ where $n \in N$ is said to have the property P if $|x_i - x_{i+1}| = n$ for atleast one i in $\{1, 2, \ldots. 2n-1\}$. Show that for each n, there are more permutations with property P than without.

20. State and prove general principle of inclusion exclusion. Using GPIE find the number of primes between 1 and 48.

21. The n sectors $n \geq 1$ of a circle are to be coloured by k distinct colours where $k \geq 3$ in such a way that each sector is coloured by one colour and any *two* adjacent sectors must be coloured by different colours. Let an denote the number of ways this can be done. (i) Evaluate a_1, a_2, a_3; (ii) Find a recurrence relation for $(a_n), n \geq 4$ and solve it.

22. (a) Let $F(n, m), n, m \in N$ denote the number of surjective mappings from N_n to N_m. Show that

    $$F(n, m) = \sum_{k=0}^{m} (-1)^k \binom{m}{k} (m-k)^n.$$

    (b) Show that $\sum_{k=0}^{m} (-1)^k \binom{m}{k} (m-k)^n = 0$ if $n < m$.

    (c) Show that $\sum_{k=0}^{n} (-1)^k \binom{n}{k} (n-k)^n = n!$.

(3 × 5 = 15)