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Reg. No.................................

Name....................................

M.Sc. DEGREE (C.S.S.) EXAMINATION, MAY 2018

Fourth Semester

Faculty of Science

Branch I (A) : Mathematics

MT 04 E13—ALGORITHMIC GRAPH THEORY

(2012 Admission onwards)

Time : Three Hours | Maximum Weight : 30

Part A

Answer any **five** *questions.*
Each question has weight 1.

1. State and prove the first theorem of graph theory.
2. When can you say that an algorithm efficient ?
3. Define rooted tree.
4. Determine all self-centered trees.
5. Define a flow in a network N.
6. Show that if G is an n-connected graph, then $G + K_1$ is $(n + 1)$-connected.
7. Briefly explain the Marriage problem.
8. In a (b, v, r, k, λ) design, prove that $bk = vr$.

$(5 \times 1 = 5)$

Part B

Answer any **five** *questions.*
Each question has weight 2.

9. Construct a graph of order 5 whose vertices have degree 1, 2, 2, 3, 4. What is the size of this graph ?
10. Write an algorithm to determine the first word from a list of n words, and to output this world and its location in the list.
11. Prove that if G is a graph with the property that G contains a unique u-v path for every two vertices u and v of G then G is a tree.

Turn over

12. Write an algorithm to determine the center of a tree T and prove that C(T) is isomorphic to K_1 or $K_{2.}$

13. Define the connection number C(G) of a graph G. Find C(K_p) and C(C_p).

14. Prove that K(G) $\leq \lambda$ (G) for any graph G.

15. Prove that for every positive integer *n,* the graph K_{2n} is 1-factorable.

16. In a (b, v, r, k, λ) design, prove that $\lambda(v-1)=r(k-1)$.

(5 × 2 = 10)

Part C

Answer any **three** *questions.*
Each question has weight 5.

17. State and prove Havel-Hakimi theorem on degree sequencies of a graph. Also check whether the sequence 4, 4, 3, 3, 2, 2 is graphical or not.

18. Write Kruskal's Algorithm and prove that it produces a minimum spanning tree in a non trivial connected weighted graph.

19. Explain Critical path algorithm. and find its complexity.

20. Let N be a network with underlying diagraph D. Prove that a flow f in N is a maximum flow if and only if there is no f-augmenting semipath in D.

21. Write an algorithm to determine the connectivity of a graph G of order P with vertex set V (G) = $\{v_1, v_2, ..., v_p\}$ find its complexity.

22. Prove that a nontrivial graph G has a 1-factor if and only if for every proper subset S of V (G), the number if odd components of G-S does not exceed $|S|$.

(3 × 5 = 15)