QP CODE: 19002355

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Reg No : ....

Name : ....

M.Sc. DEGREE (C.S.S) EXAMINATION, NOVEMBER 2019

First Semester

Faculty of Science

MATHEMATICS

Core - ME010104 - REAL ANALYSIS

2019 Admission Onwards

6A0EF4BF

Time: 3 Hours | Maximum Weight :30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. Prove that if f is monotonic on $[a, b]$ then f is of bounded variation on $[a, b]$.
2. Prove the additive property of arc lengths.
3. *Let* $f(x) = \begin{cases} 0 & \text{if } x \text{ is irrational} \\ 1 & \text{if } x \text{ is rational} \end{cases}$ *. Is f Riemann integrable on [0, 1] ?.*
4. *If* $f \in \mathscr{R}(\alpha)$ *and* $g \in \mathscr{R}(\alpha)$ *then show that* $fg \in \mathscr{R}(\alpha)$*.*
5. Define the unit step function I. Is it continuous?.
6. If $\{f_n\}$ and $\{g_n\}$ are sequences of bounded functions which converge uniformly on a set E, then prove that $\{f_n g_n\}$ converges uniformly on E.
7. State Weierstrass uniform convergence test for series of functions.
8. Under what conditions, a sequence $\{f_n\}$ of continuous functions defined on a compact set K, is convergent uniformly to a continuous function f ?
9. Define piontwise boundedness and uniform boundedness of a sequence of functions.
10. For $n = 0, 1, 2, \ldots,$ and x real, prove that $|sin\, nx| \leq n|sin\, x|$

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. Let f be defined on $[a, b]$. Then show that f is of bounded variation on $[a, b]$ if, and only if, f can be

expressed as the difference of two strictly increasing functions.

12. Prove that a vector valued function $\mathbf{f}$ is rectifiable if and only if each of its components is of bounded variation.

13. *If P* is a refinement of P then prove that U (P*, f,α) ≤ U (P, f,α).*

14. *If f is monotonic on [a, b] and if **α** is continuous on [a, b] then prove that f ∈ℛ(α).*

15. Prove that $\sum_{n=0}^{\infty} f_n(x)$ is a convergent series having a discontinuous sum, where
$f_n(x) = \dfrac{x^2}{(1+x^2)^n} \ (x \in \mathbb{R}, n = 0, 1, 2, \dots).$

16. Let α be monotonically increasing on $[a, b]$. Suppose $f_n \in \mathscr{R}(\alpha)$on $[a, b]$, for $n = 1, 2, 3, \dots$ and suppose $f_n \to f$ uniformly on $[a, b]$. Then prove that $f \in \mathscr{R}(\alpha)$ on $[a, b]$.

17. Let K be a compact metric space and let S be a subset of $\mathscr{C}(K)$. Prove that S is compact if and only if S is uniformly closed, pointwise bounded and equicontinuous.

18. If $\sum a_n, \sum b_n, \sum c_n$, converge to A, B, C, and if $c_n = a_0 b_n + \dots + a_n b_0$, prove that $C = AB$.

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. (i) State and prove additive property of total variation.
(ii) Let f be of bounded variation on $[a, b]$. Let V be defined on $[a, b]$ as $V(x) = V_f(a, x)$, if $a < x \leq b$ and $V(0) = 0$. Then prove that V and $V - f$ are increasing functions on $[a, b]$.

20. *Suppose α increases monotonically on [a,b], g is continuous and g(x) = G′(x) for a ≤ x ≤ b . Prove that* $\int_a^b \alpha(x) g(x) dx = G(b)\alpha(b) - G(a)\alpha(a) - \int_a^b G d\alpha.$

21.
Establish the existence of a real valued continuous function which is nowhere differentiable.

22. Prove that, for every interval $[-a, a]$ there exits a sequence of real polynomials P_n such that $P_n(0) = 0$ and such that $\lim_{n\to\infty} P_n(x) = |x|$ uniformly on $[-a, a]$.

(2×5=10 weightage)