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M.Sc. DEGREE (C.S.S.) EXAMINATION, MARCH 2015

First Semester

Faculty of Science

Branch I (A)—Mathematics

MT 01 C01—LINEAR ALGEBRA

(2012 Admissions)

Time : Three Hours Maximum Weight : 30

Part A

Answer any **five** *questions.*
Each question carries **1** *weight.*

1. If $V(F)$ is a vector space prove :
   (a) $ov = 0$ for $v \in V$.
   (b) $av = bv \Rightarrow a = b$ where $v \in V$, $a, b \in F$.
2. Prove or disprove :
   (a) Union of two subspaces of a vector space is also a subspace.
   (b) Intersection of two subspaces of a vector space is also a subspace.
3. Define the dual space and obtain its dimension.
4. If T is a linear transformation on a vector space satisfying $T^2 - T + I = 0$, prove T is invertible.
5. Give example for commutative ring and non-commutative rings. Prove your assertions.
6. Let k be a commutative ring with identity and let A and B be $m \times n$ matrics over k. Prove $\det(AB) = (\det A)(\det B)$.
7. Explain invariant direct sum and invariant subspace with example.
8. Let V be two-dimensional over the field F, of all real numbers, with a basis v_1, v_2. Find the characteristic roots of T given by $T(v_1) = v_1 + v_2$ and $T(v_2) = v_1 - v_2$.

$(5 \times 1 = 5)$

Turn over

Part B

Answer any **five** *questions.*
Each question carries 2 weight.

9. Find the co-ordinates of the vector $(2, 1, -6)$ of R^3 relative to the basis $(1, 1, 2)\,(3, -1, 0)\,(2, 0, -1)$.

10. The linear transformations T_1, T_2, T_3 are given by $T_1(x, y, z) = (x + y + z, x + y)$ $T_2(x, y, z) = (2x + z, x + y)$ and $T_3(x, y, z) = (2y, x)$.

    Prove that T_1, T_2, T_3 are linearly independent.

11. List the properties of the transpose of a linear transformation and prove two of them.

12. Define isomorphism. Show $T : V_2(R) \to V_2(R)$ given by $T(a, b) = (b, a)$ is an isomorphism.

13. Let K be a commutative ring with identity. Show that the determinant function on 2×2 matrics A over K is alternating and 2-linear as function of columns of A.

14. Let T be the linear operator on R^2, the matrix of which in the standard ordered basis is $\begin{pmatrix} 1 & -1 \\ 2 & 2 \end{pmatrix}$.

    Find all subspace of R^2 that are invariant under T.

15. Obtain equivalent conditions for λ to be a characteristic value of a linear operator T defined on the finite dimensional-vector space.

16. Obtain necessary and sufficient condition for a linear operator on a finite dimensional vector space to be singular.

$(5 \times 2 = 10)$

Part C

Answer any **three** *questions.*
Each question carries 5 weight.

17. (a) If α, β, γ are linearly independent show that $\alpha + \beta, \alpha - \beta, \alpha - 2\beta + \gamma$ are also linearly independent.

    (b) Let F be a field of real numbers and V be the set of all sequences $(a_1, a_2, \dots a_n \dots)$, $a_i \in F$ where equality, additional and scalar multiplication are defined component-wise. Verify that V is a Vector space over F. Further, show that $W = \left\{(a_1, a_2, \dots\dots a_n, \dots\dots) \in V / \lim_{n \to \infty} a_n = 0\right\}$ is a subspace of V.

18. (a) Find the subspace annihilated by the following functional x^4 :

$$f(x_1, x_2, x_3, x_4) = x_1 + 2x_2 + 2x_3 + x_4$$
$$g(x_1, x_2, x_3, x_4) = 2x_2 + x_4$$
$$h(x_1, x_2, x_3, x_4) = -2x_1 - 4x_3 + 3x_4$$

(b) Let $T : V \to W$ be linear where V and W are vector space over F. Show that :

(i) The range (T^t) is the annihilator of the null space of T.

(ii) Rank (T^t) = Rank T.

19. (a) If $B = \{(1, -1, 3)(0, 1, -1)(0, 3, -2)\}$ be a basis for $V_3(R)$ find its dual basis B^*.

(b) Find the matrix of a linear transformation T on $V_3(R)$ defined as :

$T(a, b, c) = (2b + c, a - 4b, 3c)$ with respect to the ordered basis $\{(1, 1, 1,)(1, 1, 0)(1, 0, 0)\}$.

20. State and prove the properties of determinants.

21. (a) Differentiate between simultaneous triangulation and simultaneous diagonalisation with examples.

(b) Explain annihilatory polynomial and characteristic polynomial.

22. State and prove Cayley-Hamilton theorem for linear operators.

$(3 \times 5 = 15)$