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Reg. No.................................

Name.....................................

M.Sc. DEGREE (C.S.S.) EXAMINATION, JANUARY/FEBRUARY 2017

First Semester

Faculty of Science

Branch I (a)—Mathematics

MT 01 C02—BASIC TOPOLOGY

(2012 Admission onwards)

Time : Three Hours

Maximum Weight : 30

Part A

Answer any **five** *questions.*
Each question carries 1 *weight.*

1. (a) Define co-countable topology.

   (b) Closure and interior of a set.

2. Determine the topology induced by a discrete metric on a set.

3. Define quotient topology.

4. Define projection maps. Show that projections are continuous.

5. Give an example of a connected closed subset C of R^2 such that $R^2 - C$ has infinitely many components.

6. Define a locally connected space. Give an example of a space which is connected by not locally connected.

7. Define T_1, T_2 and T_3 spaces.

8. Define a completely regular space.

$(5 \times 1 = 5)$

Part B

Answer any **five** *questions.*
Each question carries 2 *weight.*

9. Prove that if a space is second countable then every open cover of it has a countable subcover.

10. Prove that a subset of a topological space is open if and only if it is a neighbourhood of each of its points.

11. Prove that composition of continuous functions are continuous.

12. Prove that every continuous image of a compact space is compact.

Turn over

13. Show that closure of a connected subset is connected.

14. Show that every quotient space of a locally connected space is locally connected.

15. Show that in a Hausdorff space, limits of sequences are unique.

16. Show that regularity is a hereditary property.

$(5 \times 2 = 10)$

Part C

Answer any **three** *questions.*
Each question carries 5 *weight.*

17. (a) Prove that metrisability is a hereditary property.

 (b) Find out fence subsets of discrete, indiscrete and co-finite spaces.

 (c) Prove that for a subset A of X, $\text{int}(A) = X - \overline{(X - A)}$.

18. (a) Prove that in the case of finite products, the product topology is the weak topology determined by the projection functions.

 (b) Prove that every continuous real valued function on a compact space is bounded and attains its extrema.

19. (a) Let X and Y be topological spaces and $x \in X$, $f : X \to Y$ be a function, suppose X is first countable. Prove that f is continuous if and only if for every sequence $\{x_n\}$ in X which converges to $x \in X$, $\{f(x_n)\}$ converges to $f(x)$ in Y.

 (b) Let $f : X + Y$ be a function, prove that if f is continuous, then the graph G of f is homeomorphic to X.

20. Prove the following :—

 (a) Components are closed sets.

 (b) Any two distinct components are mutually disjoint.

 (c) Every non-empty connected subset is contained in a unique component.

 (d) Every space is the disjoint union of its components.

21. (a) Show that every regular Lindeloff space is normal.

 (b) Prove that a continuous bijection from a compact space onto a Hausdorff space is a homeomorphism.

22. (a) Prove that a subset of R is connected if and only if it is an interval.

 (b) Prove that an open subspace of a locally connected space is locally connected.

$(3 \times 5 = 15)$