G 1993 (Pages : 2) Reg. No.................................

Name.....................................

M.Sc. DEGREE (C.S.S.) EXAMINATION, MARCH 2015

First Semester

Faculty of Science

Branch I (A)—Mathematics

MT 01 C03—MEASURE THEORY AND INTEGRATION

(2012 Admissions)

Time : Three Hours Maximum Weight : 30

Part A (Short Answer Type Questions)

Answer any **five** *questions.*
Each question has 1 *weight.*

1. Show that the translate of a measurable set is also measurable.
2. Prove that the product of measurable functions is also measurable.
3. Establish the monotonicity property of Lebesgue integral.
4. Explain : (a) $f = g\,(a \cdot e)$; (b) Borel set.
5. Define : (a) Counting measure ; (b) Dirac measure.
6. Establish the uniqueness assertion of the Jordan Decomposition Theorem.
7. If $f_n \to f$ in measure, show that there is a subsequence $\left\{f_{n_k}\right\}$ which converges to f *a.e.*
8. Define : (a) Measurable rectangle ; (b) Complete measure.

$(5 \times 1 = 5)$

Part B (Short Essay Type Questions)

Answer any **five** *questions.*
Each question has 2 *weight.*

9. Show that outer measure is countably sub-additive.
10. Prove or disprove : Composition of measurable functions is also measurable.
11. Define passage of the limit under the integral sign and show that point-wise convergence alone is not sufficient to justify passage of the limit under the integral sign.
12. State Fatou's lemma. Give an example to show that we may have strict inequality in the Fatou's Lemma.

Turn over

13. Explain : finite measure, sigma-finite measure, saturated measure, completion of a measure space.
14. Establish the uniqueness of the function f in the Radon-Nikodym theorem.
15. Prove the 'Completeness' theorem for convergence in measure.
16. By integrating e^{-xy} sin $2y$ with respect to x and y show that

$$\int_0^\infty e^{-y} (\sin 2y) \,/\, y \, dy = \text{are tan } z.$$

(5 × 2 = 10)

Part C (Long Essay Type Questions)

Answer any **three** *questions.*
Each question has 5 *weight.*

17. (a) List the properties of outer measure of a set and prove two of them.
    (b) Establish Vitali's theorem on non-measurable set.
18. (a) Define Riemann integral and Lebesgue integral. Also discuss the Principal short comings of the Riemann integral.
    (b) Establish the linearity and monotonicity of Lebesgue integration of measurable non-negative functions.
19. (a) State and prove General Lebesgue. Dominated Convergence theorem.
    (b) State and prove the Vitali Covering lemma.
20. State and prove the Radon Nikoydm theorem.
21. (a) Establish the extension theorem.
    (b) Show that the measurability of functions is preserved under the formation of point-wise limits.
22. (a) Obtain necessary and sufficient condition for the rectangle A × B to b non-measurable.
    (b) State and prove Fubini's theorem.

(3 × 5 = 15)