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# M.Sc. DEGREE (C.S.S.) EXAMINATION, MARCH 2015

## First Semester

Faculty of Science

Branch I–(A) Mathematics

MTO 1C 04—GRAPH THEORY

(2012 Admissions)

Time : Three Hours Maximum Weight : 30

### Part A

*Answer any* **five** *questions.*
*Each question has 1 weight.*

1. Define a strong digraph and a symmetric digraph with examples.
2. What do you mean by the wheel $w_n$ ? Draw $w_6$ and explain.
3. Prove that, for a connected graph G we have $r$ (G) ≤ diam (G) 2 $r$ (G).
4. Define (i) signed graph ; (ii) balanced graph.
5. Define Eulerian and Hamiltonian graphs. Draw a graph which is both Eulerian and Hamiltonian.
6. Characterise an independent set.
7. Explain the timetable problem and convert this problem into a graph-theoretic one.
8. Draw Petersen graph and show that it is not non-planar.

(5 × 1 = 5)

### Part B

*Answer any* **five** *questions.*
*Each question has 2 weight.*

9. Define a line graph and list its properties.
10. Characterise a cut edge. Also give example for blocks.
11. Obtain the relation between the number of vertices and number of edges of any tree.
12. Explain three particular cases of the connector problem.
13. Define Hamiltonian connected graph. Also given an example of graph which is Hamiltonian connected and another graph which is not Hamiltonian connected.
14. Prove : In a critical graph G, no vertex cut is a clique.

Turn over

15. Show that a Hamiltonian cubic graph is 3-edge chromatic.

16. Show that $k_5$ is non-planar.

$(5 \times 2 = 10)$

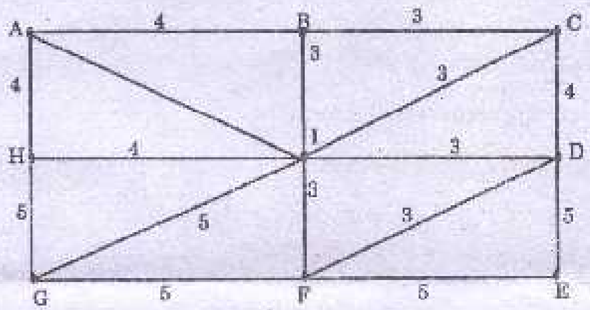
## Part C

*Answer any* **three** *questions.*
*Each question has 5 weight.*

17. (a) Draw graphs with edge connectivity 1, 2 and 3. Also obtain the maximum value of edge connectivity.

(b) Show that a graph with at least two vertices is bipartite if and only if it contains no odd cycles.

18. Use Kruskal's algorithm to find the shortest spanning tree of the graph given below :




19. (a) Establish Jordan's theorem on the centre of a tree.

(b) Obtain the recursive formula for the number of spanning tree for a connected labelled graph.

20. (a) Establish equivalent conditions for a connected graph to be Eulerian.

(b) Establish Dirac result on Hamiltonian graph.

21. (a) Establish Konig's theorem on chromatic index of a graph.

(b) State and prove Heawood five-color theorem.

22. (a) Prove that a graph is planar if and only if it is embeddable on a sphere.

(b) Prove or disprove the converse of Konigtheorem on the chromatic index.

(c) Give examples for (i) plane graph ; (ii) planar graph ; (iii) dual of a plane graph ; (iv) independent set.

$(3 \times 5 = 15)$