F 7736 (Pages : 3) Reg. No.................................

Name.................................

M.Sc. DEGREE (C.S.S.) EXAMINATION, FEBRUARY 2014

First Semester

Faculty of Science

Branch I (A)—Mathematics

MTO IC 04—GRAPH THEORY

(2012 Admission onwards)

Time : Three Hours Maximum Weight : 30

Part A

Answer any **five** *questions.*
Each question has weight 1.

1. Show that the sum of the degrees of the vertices of a graph is equal to twice the number of its edges.
2. For any simple graph G, show that $\Gamma(G) = \Gamma(G^C)$.
3. Give an example of a graph with n vertices and $n-1$ edges that is not a tree.
4. Prove that if $m(G) = n(G)$ for a simple connected graph G, then G is unicyclic.
5. Show that any critical graph G is connected.
6. Determine the value of the parameters $\alpha, \alpha', \beta, \beta'$ for the Petersen graph P.
7. Prove that the Petersen graph P is non-planar.
8. Show that the complement of a simple planar graph with 11 vertices is non-planar.

$(5 \times 1 = 5)$

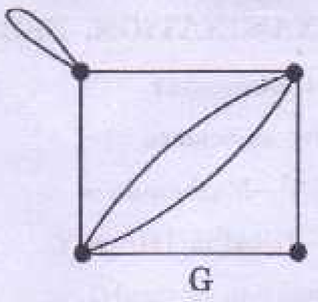
Part B

Answer any **five** *questions.*
Each question has weight 2.

9. If the simple graphs G_1 and G_2 are isomorphic, show that $L(G_1)$ and $L(G_2)$ are isomorphic.
10. Prove that an edge $e = xy$ is a cut edge of a connected graph G if and only if there exists vertices u and v such that e belongs to every $u - v$ path in G.

Turn over

11. Find the number of spanning trees $\tau(G)$ for the following graph G.



G

12. Prove that every connected graph contains a spanning tree.

13. Prove that the n-cube Q_n is Hamiltonian for every $n \geq 2$.

14. Show that for any graph G with n vertices and independence number α, $\frac{n}{\alpha} \leq \chi \leq n - \alpha + 1$.

15. Prove that K_5 is non-planar.

16. Prove that every Planar graph is 6 vertex colorable.

(5 × 2 = 10)

Part C

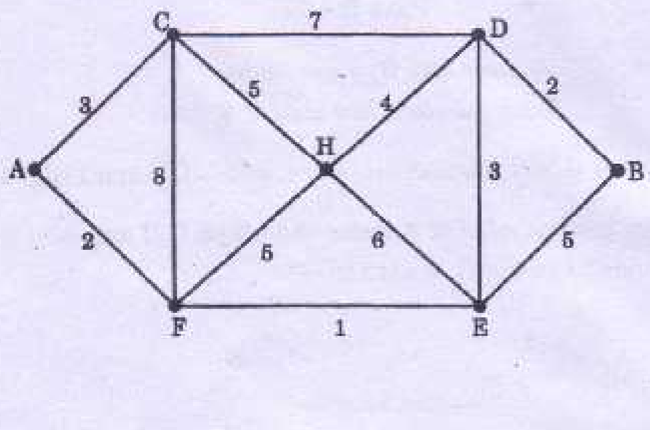
Answer any **three** *questions.*
Each question has weight 5.

17. (a) Prove that in a 2-connected graph G, any two longest cycles have atleast two vertices in common.

    (b) Show that a graph G with atleast three vertices is 2-connected if and only if any two edges of G lie on a common cycle.

18. (a) Describe Dijkstra's algorithm.

    (b) Find the shortest path from A to B in the graph given below :



19. For the graph G determine two distinct minimum-weight spanning trees using :

    (a) Kruskal's algorithm. (b) Prim's algorithm.

    What is the weight of such a tree ?

20. If a connected graph G is neither an odd cycle nor a complete graph, then show that $\chi(G) \leq \Delta(G)$.

21. (a) Prove that $\chi'(k_n) = \begin{cases} n-1 & \text{if } n \text{ is even} \\ n & \text{if } n \text{ is odd.} \end{cases}$

    (b) Show that a simple cubic graph with a cut edge is 4-edge-Chromatic.

22. (a) Let G be a simple graph with $n \geq 3$ vertices. If for every pair of non-adjacent vertices u, v of G, $d(u) + d(v) \geq n$. Prove that G is Hamiltonian.

    (b) Show that if a cubic graph G has a spanning closed trial, then G is Hamiltonian.

$(3 \times 5 = 15)$