F 5536 (Pages : 2) Reg. No.....

Name.....

M.Sc. DEGREE (C.S.S.) EXAMINATION, FEBRUARY 2016

First Semester

Faculty of Science

Branch I (A)—Mathematics

MT 01 C 04—GRAPH THEORY

(2012 Admission onwards)

Time : Three Hours Maximum Weight : 30

Part A

Answer any **five** *questions.*
Each question carries a weight of 1.

1. Define intersection and join of two graphs with illustrations.
2. Obtain a sufficient condition for a wheel to be connected.
3. Give an example of (i) a tree with two centroidal vertices, one of which is also a central vertex ; (ii) A tree with disjoint center and centroid.
4. List the properties of trees.
5. Define Eulerian and Hamiltonian graphs. Draw a graph which is neither Eulerian nor Hamiltonian.
6. Draw Peterson graph and show that it is not 1–factorable.
7. Draw a disconnected graph and its connected dual.
8. Show that every planar graph is 6-vertex colorable.

$(5 \times 1 = 5)$

Part B

Answer any **five** *questions.*
Each question carries a weight of 2.

9. Explain edge connectivity and block with examples.
10. Show that every tournament contains a diverted Hamiltonian path.
11. Prove that for a simple connected graph G, L(G) is isomorphic to G if and only if G is a cycle.
12. Examine whether a signed tree is balanced.
13. For any graph G for which $\delta > 0$ show that $\alpha^1 + \beta^1 = n$.
14. Explain : Knight's tour in a chess board.

Turn over

15. Show that the Petersen graph is 4-edge-chromatic.

16. Show that $k_{3,3}$ is non-planar.

(5 × 2 = 10)

Part C

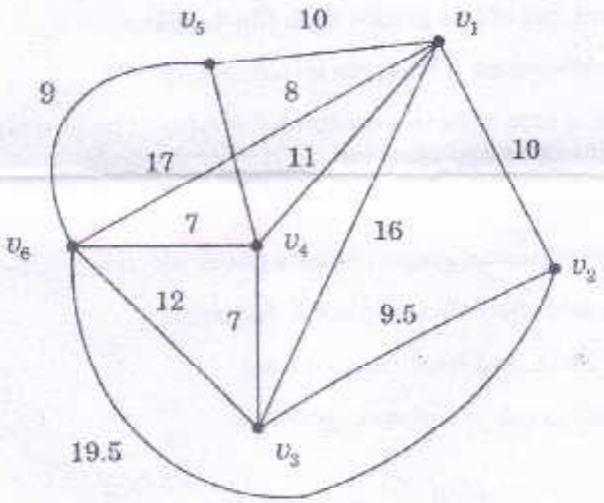
Answer any **three** *questions.*
Each question carries a weight of 5.

17. (a) Obtain necessary and sufficient condition for the edge connectivity of a graph to be K.

    (b) Show that the set of automorphisms of a simple graph is a group under a suitable binary operation.

18. Establishing the required lemmas, obtain Cayley's formula for the number of spanning trees of a labelled complete graph.

19. Obtain the shortest spanning tree for the graph given below using Prim's algorithm.



20. Define :

    (a) Maximum and maximal matching.

    (b) Vertex-independent and edge-independent sets.

    (c) Maximum and maximal independent sets.

    (d) Independence number and covering number.

    (e) Vertex coloring and Vertex covering.

21. (a) Determine the chromatic index of the complete graph.

    (b) Establish Euler's formula and derive two of its consequences.

22. State and prove Vizing-Gupta theorem on edge-coloring of graphs.

(3 × 5 = 15)