E 803 (Pages : 3) Reg. No..............................

Name..................................

B.Sc. DEGREE (CBCSS) EXAMINATION, NOVEMBER 2010

Third Semester

Complementary Course— Mathematics—Vector Calculus

FOURIER SERIES AND ANALYTIC GEOMETRY

(Common for Physics, Chemistry, Geology, Petrochemicals, etc.)

Time : Three Hours Maximum Weight : 25

Part A (Objective Type Questions)

Answer **all** *questions.*
Each bunch of four questions has weight 1.

I. 1 Define a unit vector.

2 If $\bar{a} = [1,3,2], \bar{b} = [2,0,-5]$, then $\bar{a} \cdot \bar{b} = ...$

3 Write the parametric representation of the line through (4, 2, 0) in the direction $\bar{i} + \bar{j}$.

4 If $\bar{r}(t) = a\cos t\, \bar{i} + a \sin t\, \bar{j}$, describes the motion of a particle, then its speed is . . .

II. 5 Find a vector normal to the surface $x^2 + y^2 + z^2 = 1$ at $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$.

6 If $f(x, y, z) = x^2 + y^2 + z^2$, then div (grad f) = . . .

7 The directional derivative of f in the direction of the unit vector $\bar{b}$ is the dot product . . .

8 If $\bar{V} = [-y, x, 0]$, then curl $\bar{V} = ...$

III. 9 If a_n is the coefficient of $\cos nx$ in the Fourier series of a periodic function $f(x)$ of period 2π then for $n = 1, 2, ..., a_n = ...$

10 What is the Fourier series of an odd function $f(x)$ of period 2π.

11 Write the general form of Legendre's differential equation.

12 Define the Gamma function.

IV. 13 Write the equations of the asymptotes of the hyperbola $x^2 - y^2 = 1$.

14 Write the Cartesian equation of the curve given by $x = 4 \cos t$, $y = 4 \sin t$, $0 \le t \le 2\pi$.

15 Write all the polar co-ordinates of the point $P\left(2, \frac{\pi}{3}\right)$.

16 Write the polar equation of a parabola with directrix $x = 1$.

$(4 \times 1 = 4)$

Turn over

Part B (Short Answer Type Questions)

Answer any **five** *questions.*
Each question has weight 1.

17 Find the directional derivative of $f(x, y, z) = 2x^2 + 3y^2 + z^2$ at the point (2, 1, 3) in the direction of $\bar{a} = -\bar{i} + 2\bar{j} + 2\bar{k}$.

18 For any twice continuously differentiable scalar function show that curl (grad f) = 0.

19 Find the work done by the force $\overline{F} = \left[y^2, -x^2\right]$ in the displacement along the line segment from (0, 0) to (1,4).

20 State Green's theorem in the plane.

21 Sketch the graph of the function $f(x) = |x|, -\pi \le x \le \pi$ and its periodic with period 2π.

22 State the Rodrigue's formula for Legendre polynomial $P_n(x)$. Using this find $P_1(x)$.

23 Find the foci of the ellipse $9x^2 + 16y^2 = 135$.

24 Convert the polar equation $r\cos\left(\theta - \frac{\pi}{4}\right) = \sqrt{2}$ to Cartesian form.

(5 × 1 = 5)

Part C (Short Essay Questions)

Answer any **four** *questions.*
Each question has weight 2.

25 Show that $\overline{V} = [2x, 4y, 8z]$ can be expressed as the gradient of a scalar function.

26 Evaluate surface integral $\iint_S \overline{F} \cdot \bar{n}\, dA$ using divergence theorem, where $\overline{F} = x^3\bar{i} + x^2y\,\bar{j} + x^2z\,\bar{k}$ and S is the surface of the cub $|x| \le 1, |y| \le 1, |z| \le 1$.

27 Find the Fourier series of the function $f(x) = x + \pi$ if $-\pi \le x \le \pi$ and $f(x + 2\pi) = f(x)$.

28 Show that $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \sin x$.

29 Find the Cartesian equation of the hyperbola centred at the origin that has a focus at (4, 0) and the line $x = 2$ as the corresponding directrix.

30 Find the Cartesian equivalents of the polar equations (i) $r(2\cos\theta - \sin\theta) = 4$; (ii) $r = 1 - \cos\theta$.

(4 × 2 = 8)

Part D (Essay Questions)

Answer any **two** *questions.*
Each question has weight 4.

31 State Stokes' theorem (without proof). Verify the theorem for $\overline{F} = [y, z, x]$ and S is the paraboloid

$z = f(x, y) = 1 - x^2 - y^2, z \geq 0.$

32 Find the Fourier series of the function

$$f(x) = \begin{cases} -k, & -\pi < x < 0 \\ k & 0 \leq x < \pi \end{cases}$$

and $f(x + 2\pi) = f(x)$

Also sketch the graph of $f(x)$.

33 A wheel of radius a rolls along a horizontal straight line. Find parametric equations for the path traced by a point P on the wheel's circumference. Indicate the portion of the graph traced by P and the direction of motion.

$(2 \times 4 = 8)$