E 2124 (Pages : 4) Reg. No.....

Name.....

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, MAY 2015

Second Semester

Complementary Course—Mathematics

INTEGRAL CALCULUS AND MATRICES

(Common for B.Sc. Physics, Chemistry, Petrochemicals, Geology, Food Science and Quality Control and Computer Maintenance and Electronics)

[2013 Admission onwards]

Time : Three Hours Maximum : 80 Marks

Part A

Answer **all** *questions.*

Each question carries 1 *mark.*

1. Suppose $\int_{-3}^{0} g(t)\,dt = \sqrt{2}$. Find $\int_{-3}^{0} \frac{g(r)}{\sqrt{2}}\,dr$.
2. State mean value theorem for definite integrals.
3. Solve the initial value problem :

   $\frac{ds}{dt} = \cos t + \sin t, s(\pi) = 1$.

4. Find the antiderivative of $\frac{5}{x^2}$.
5. Find $\frac{d}{dx}\int_{0}^{x} \frac{1}{1+t^2}\,dt$.
6. Write the formula for the length of a smooth curve $x = g(y), c \le y \le d$.
7. Write the surface area formula for revolution about y-axis, a smooth curve $x = g(y) \ge 0$ on $[c, d]$.
8. State first form of Fubini's theorem.
9. What is a matrix polynomial ?
10. Find the eigen value of $\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$.

(10 × 1 = 10)

Turn over

Part B

Answer any **eight** *questions.*
Each question carries 2 marks.

11. Suppose h is continuous and $\int_{-1}^{1} h(r)\,dr = 0$ and $\int_{-1}^{3} h(r)\,dr = 6$. Find $\int_{1}^{3} h(u)\,du$.

12. Evaluate $\int \sqrt{1+y^2} \cdot 2y\,dy$.

13. Find $\int x^2 \sin\left(x^3\right) dx$.

14. Find the area of the region between $y = 2x$ and the x-axis on the interval $[0,b]$.

15. Show that the value of $\int_{0}^{1} \sqrt{1+\cos x}\,dx$ cannot be possible.

16. Can a function have more than one antiderivative. If so how are the antiderivatives related ?

17. Find $\int_{\pi}^{2\pi} \int_{0}^{\pi} (\sin x + \cos y)\,dx\,dy$.

18. Find the area of the region R enclosed by the parabola $y = x^2$ and the line $y = x + 2$.

19. Find the characteristic equation of :

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & 2 \\ 0 & 0 & 7 \end{bmatrix}.$$

20. Reduce $\begin{bmatrix} 1 & 0 & 2 & 3 \\ 2 & 1 & 0 & 1 \\ 4 & 1 & 4 & 7 \end{bmatrix}$ to the normal form.

21. Verify Cayley-Hamilton theorem for the matrix $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$.

22. Show that A^T has the same eigen values as A.

$(8 \times 2 = 16)$

Part C

Answer any **six** *questions.*
Each question carries 4 *marks.*

23. Find the average height of the paraboloid $z = x^2 + y^2$ over the square $0 \le x \le 2, 0 \le y \le 2$.

24. Evaluate $\int_0^1 \int_0^{\pi} \int_0^{\pi} y \sin z \, dx \, dy \, dz$.

25. Evaluate $\int \frac{3x+2}{\sqrt{1-x^2}} \, dx$.

26. Find the area of the region between the x-axis and the graph of $f(x) = x^3 - x^2 - 2x, -1 \le x \le 2$.

27. Find the area of the regions enclosed by the curves $x^3 - y = 0$ and $3x^2 - y = 4$.

28. Find the volume of the solid generated by revolving the regions bounded by the lines :

    $x = 0, y = -1, y = 1$ and the curve $x = \sqrt{5}\, y^2$ about y-axis.

29. Find the length of the curve :

    $x = \frac{y^{3/2}}{3} - y^{1/2}$ from $y = 1$ to $y = q$.

30. Calculate A^4 using Cayley-Hamilton theorem if, $A = \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$.

31. Show that the following system of equations are inconsistent :

$$\begin{aligned} x_1 + x_2 + 2x_3 - x_4 &= 5 \\ 2x_1 + 3x_2 - x_3 - 2x_4 &= 2 \\ 4x_1 + 5x_2 + 3x_3 &= 7. \end{aligned}$$

$(6 \times 4 = 24)$

Part D

Answer any **two** *questions.*
Each question carries 15 *marks.*

32. (i) A pyramid 3 m high has a square base that is 3 m on a side. The cross-section of the pyramid perpendicular to the attitude x_m down from the vertex is a square x_m on a side. Find the volume of the pyramid.

    (ii) Find the area of the surface generated by revolving the curve : $y = x^3, 0 \le x \le \frac{1}{2}$, about the x-axis.

33. (i) Find the polar moment of inertia about the origin of a thin plate of density $\delta(x, y) = 1$ bounded by the quarter circle $x^2 + y^2 = 1$ in the first quadrant.

(ii) Evaluate $\int_0^2 \int_0^{\sqrt{4-y^2}} \left(x^2 + y^2\right) dx\, dy$.

34. Find the volume of the region D enclosed by the surfaces $z = x^2 + 3y^2$ and $z = 8 - x^2 - y^2$.

35. (i) Solve the following system by determinants :

$$2x - 5y + 2z = 2$$
$$x + 2y - 4z = 5$$
$$3x - 4y - 6z = 1.$$

(ii) Obtain the row-equivalent canonical matrix of :

$$\begin{bmatrix} 1 & 1 & 1 & 2 \\ 2 & 1 & -3 & -6 \\ 3 & -3 & 1 & 2 \end{bmatrix}.$$

(2 × 15 = 30)