E 5648 (Pages : 4) Reg. No.................................

Name.....................................

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, MAY 2013

Second Semester

INTEGRAL CALCULUS AND MATRICES

(Complementary Course to Physics/Chemistry/Petrochemicals/Geology/Food Science and Quality Control and Computer Maintenance and Electronics)

Time : Three Hours Maximum Weight : 25

Part A

Answer **all** *questions.*
A bunch of **four** *questions has weight* 1.

I. 1. Express $\lim_{\|p\| \to 0} \sum_{k=1}^{n} 2C_k^{\,3}\, \Delta x_k$, where p is a partition of $[-1, 0]$ as a definite integral.

2. State mean value theorem for definite integrals.

3. Find the average value of $f(x) = x^2 - 1$ on $\left[0, \sqrt{3}\right]$.

4. Find $\int_{-4}^{4} |x|\, dx$.

II. 5. Find $\int_{\pi/4}^{\pi/2} \cot\theta \operatorname{cosec}^2\theta \, d\theta$.

6. Find the area between the curves $y = \sec^2 x$ and $y = \sin x$ from 0 to $\pi/4$.

7. Find the volume of the solid generated by revolving the region between the $y-$ axis and the curve $x = \frac{2}{y}, 1 \le y \le 4$, about the $y-$ axis.

8. Find the formula for calculating the length of a smooth curve $x = g(y), c \le y \le d$.

III. 9. Give an example of a smooth curve.

10. State first form of Fubini's theorem.

11. Find the area enclosed by lemniscate $y^2 = 4\cos 2\theta$.

Turn over

12. Sketch the region of integration of $\int_1^2 \int_y^{y^2} dx\, dy$.

IV. 13. Find the rank of $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 0 & 2 & 2 \end{bmatrix}$.

14. Write the eigen values of $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$.

15. If the eigen values of a matrix A are $\lambda_1, \lambda_2 ... \lambda_n$. Then what is the eigen value of A^2.

16. Find the characteristic polynomial of $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$.

$(4 \times 1 = 4)$

Part B

Answer any **five** *questions.*
Each question has weight 1.

17. Suppose that $f(x) = \frac{d}{dx}\left[1 - \sqrt{x}\right]$ and $g(x) = \frac{d}{dx}(x+2)$. Find $\int [f(x) - g(x)]\, dx$ and $\int (x + f(x))\, dx$.

18. Find $\frac{dy}{dx}$ if $y = \int_1^{x^2} \cos t\, dt$.

19. Find the volume of the solid generated by revolving the region bounded by $y = \sqrt{x}$ and the lines $y = 1$, $x = 4$ about the line $y = 1$.

20. A curved wedge is cut from a cylinder of radius 3 by two planes. One plane is perpendicular to the axis of the cylinder. The second plane crosses the first plane at 45° angle at the center of the cylinder. Find the volume of the wedge.

21. Calculate $\iint_R \frac{\sin x}{x}\, d\,A$, where R is the triangle in the xy plane bounded by the x - axis, the line $y = x$ and the line $x = 1$.

22. Find the limits of integration for integrating a function $f(r, \theta)$ over the region R that lies inside the cardioid $r = 1 + \cos\theta$ and outside the circle $r = 1$.

23. By reducing to the normal form find the rank of $\begin{bmatrix} 3 & 1 & 2 & 5 \\ -1 & 4 & 1 & -1 \\ 1 & 9 & 4 & 3 \end{bmatrix}$.

24. If $A = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$. Find the inverse of A.

(5 × 1 = 5)

Part C (Short Essays)

Answer any **four** *questions.*
Each question has weight 2.

25. Use a definite integral to find the area of the region between $y = 2x$ and the x - axis on the interval $[0, b]$.

26. Find the area of the region enclosed by $y = 2\sin x$ and $y = \sin 2x$, $0 \le x \le \pi$.

27. Find the length of the curve $y = \frac{4\sqrt{2}}{3}x^{3/2} - 1$, $0 \le x \le 1$.

28. Evaluate $\int_0^1 \int_0^{\sqrt{1-s^2}} 8t\, dt\, ds$ in the st plane.

29. Change the Cartesian integral into an equivalent polar integral and evaluate
$$\int_{-1}^{1} \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} \left(x^2 + y^2\right) dx\, dy.$$

Turn over

30. Show that the following system of equations are incosistent :

$$u - 2v + w - t + 1 = 0$$
$$3u - 2w + 3t + 4 = 0$$
$$5u - 4v + t + 3 = 0.$$

(4 × 2 = 8)

Part D (Essay Type)

Answer any **two** *questions.*
Each question has weight 4.

31. Find the volume of the solid generated by revolving the region in the first quadrant bounded above by the line $y = \sqrt{2}$, below by the curve $y = \sec x \tan x$ and on the left by the y-axis, about the line $y = \sqrt{2}$.

32. Find the volume of the region D enclosed by the surface $z = x^2 + 3y^2$ and $z = 8 - x^2 - y^2$.

33. Using Cayley-Hamilton theorem S.T. $A^3 - 6A^2 + 11A - 6I = 0$ where $A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 2 \\ -1 & 1 & 3 \end{bmatrix}$ and hence find A^{-1}.

(2 × 4 = 8)