E 5310 (Pages : 3) Reg. No.....

Name.....

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, APRIL 2013

Fourth Semester

FOURIER SERIES, DIFFERENTIAL EQUATIONS, NUMERICAL ANALYSIS AND ABSTRACT ALGEBRA

(Complementary Course to Physics, Chemistry, Petrochemicals, Geology, Food Science and Quality Control and Computer Maintenance and Electronics)

[2011 Admission onwards]

Time : Three Hours Maximum Weight : 25

Part A

Answer **all** *questions.*
Each bunch of **four** *questions has weight* 1.

I. 1 What is fundamental period of a function $f(x)$?

2 Write the Euler formula for Fourier coefficients.

3 Define a power series.

4 What is Legendre's polynomial of degree n.

II. 5 Write the parametric equations of a surface.

6 What are direction cosines ?

7 Write the partial differential equation representing the set of all spheres with centers on the z-axis.

8 Consider the equation $x\frac{dz}{dx} = z^2 + x^2$. Is this equation linear ?

III. 9 Round off the number 81.255 to two decimal places.

10 Define the term absolute error.

11 Find the Maclaurin series expansion of e^x.

12 Write Newton-Raphson formula for approximation.

IV. 13 Let * be defined on $2z - \{2n \mid n \in z\}$ by letting $a * b = a - b$. Determine whether the binary operation * gives a group structure on $2z$.

14 Write a non-trivial proper subgroup of Z_4.

Turn over

15 If $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 2 & 5 & 3 & 1 \end{pmatrix}$ $\tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 5 & 4 & 2 & 1 \end{pmatrix}$ Find $\sigma\tau$

16 Define a ring homomorphism.

$(4 \times 1 = 4)$

Part B

Answer any **five** *questions.*
Each question has weight 1.

17 Sketch the graph of $f(x) = |x|$ in the interval $-\pi < x < \pi$.

18 Write the formula for the Bessel functions $J_0(x)$ and $J_1(x)$.

19 Find the condition that the plane $lx + my + nz + p = 0$ should touch the central Conicoid $ax^2 + by^2 + cz^2 = 1$.

20 Eliminate the arbitrary function f from the equation $z = xy + f(x^2 + y^2)$.

21 Calculate the value of $\sqrt{102} - \sqrt{101}$ correct to four significant figures.

22 Obtain the range of values within which the exact value of $\frac{1.265\,(10.21 - 7.54)}{47}$ lies, if all the numerical quantities are rounded-off.

23 Describe all the elements in the cyclic subgroup of GL (2, R) generated by the matrix $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

24 Determine whether the given set of vectors is a basis for R^3 over R.

$[(-1, 1, 2), (2, -3, 1), (10, -14, 0)]$

$(5 \times 1 = 5)$

Part C

Answer any **four** *questions.*
Each question has weight 2.

25 Find the Fourier series for the function $f(x) = \begin{cases} -2x \text{ if } -\pi < x < 0 \\ 2x \text{ if } 0 < x < \pi \end{cases}$.

26 Solve the equation $y' - y = 0$. using power series method.

27 Find the integral curves of the equations $\frac{dx}{x+z} = \frac{dy}{y} = \frac{dz}{z+y^2}$.

28 Use bisection method to obtain a root correct to 3 decimal places for the equation $x^3 - x^2 - 1 = 0$.

29 Use newton-Raphson method to obtain a root of $x^3 + 3x^2 - 3 = 0$ correct to 3 decimal places.

30 If G is a group with binary operation * and if a and b are any elements of G, then show that the linear equation $a * x = b$ and $y * a = b$ have unique solutions x and y in G.

$(4 \times 2 = 8)$

Part D

Answer any **two** *questions.*
Each question has weight 4.

31 Find the Fourier series of the periodic function $f(x) = 1 - x^2, -1 < x < 1$ and sketch $f(x)$.

32 Find the general integrals of the linear partial differential equation $y^2 p - xyq = x(z - 2y)$.

33 Use quotient-difference method to obtain the approximate roots of the equation $x^3 - x^2 - 2x + 1 = 0$.

$(2 \times 4 = 8)$