

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, MARCH 2017**Fourth Semester****Complementary Course—Mathematics****FOURIER SERIES, DIFFERENTIAL EQUATIONS, NUMERICAL ANALYSIS AND
ABSTRACT ALGEBRA**

[For the programme B.Sc. Physics/Chemistry/Petrochemicals/Geology/Food Science and
Quality Control and Computer Maintenance and Electronics]

[2013 Admission onwards]

Time : Three Hours

Maximum Marks : 80

Part A

Answer all questions.

Each question carries 1 mark.

1. Check whether $f(x) = |x^3|$ is an odd function.
2. Write the Bessel function of the second kind of order ν .
3. Write the partial differential equation representing the family of concentric circles.
4. Write the Newton-Raphson formula.
5. Write the Rodrigue's formula.
6. Find the cube root of 2 correct to three decimals.
7. If $y = 4x^3 - 0.16x$. Find the percentage error in y at $x = 0.5$, if the error in x is 0.35.
8. Find the order of the element 2 in the group \mathbb{Z}_4 .
9. Define homomorphism. How many homomorphisms are there from \mathbb{Z}_2 onto \mathbb{Z}_8 .
10. Give a basis for $\mathbb{Q}(\sqrt{3})$ over \mathbb{Q} .

(10 × 1 = 10)

Turn over

Part B*Answer any eight questions.**Each question carries 2 marks.*

11. Find the Fourier series for the function $f(x) = 3x(\pi^2 - x^2)$ ($-\pi < x < \pi$).
12. Eliminate the function from $z = f\left(\frac{xy}{z}\right)$.
13. Form a partial differential equation by eliminating a and b from $2z = (ax + y)^2 + b$.
14. Find the integral curves of the equation $\frac{dy}{cy - bz} = \frac{dy}{az - cx} = \frac{dz}{bx - ay}$, show that they are circles.
15. Evaluate $\sqrt{3} + \sqrt{11}$ correct to 4 decimal places.
16. Solve the equation $x^3 - 9x + 1 = 0$ for the root lying between 2 and 3, correct to three significant digits.
17. Find a real root of the equation $f(x) = x^3 - 3x^2 + 5x - 10$ using bisection method.
18. Find the $\sqrt[3]{23}$ correct to 4 decimal places by Newton's method.
19. Prove that order of an element in a finite group divides the order of the group.
20. Find all the subgroups of \mathbb{Z}_{12} .
21. Find the order of each element in $\mathbb{Z}_8 \oplus \mathbb{Z}_4$.
22. Prove that every subgroup of a cyclic group is cyclic.

 $(8 \times 2 = 16)$ **Part C***Answer any six questions.**Each question carries 4 marks.*

23. A function $f(x)$ is defined with in the range $(0, 2\pi)$ by $f(x) = \begin{cases} x & , 0 < x < \pi \\ 2\pi - x & , \pi < x < 2\pi \end{cases}$ Express $f(x)$ as a Fourier series in $(0, 2\pi)$.
24. Solve $x^2p + y^2q = nxy$.

25. Show that $x^3 = \frac{2}{5} P_3(x) + \frac{3}{5} P_1(x)$.
26. Show that $nP_n(x) - xP'_n = P'_{n-1}$.
27. Express $J_4(x)$ in terms of $J_0(x)$ and $J_1(x)$.
28. Using Newton-Raphson method, find correct to four decimals the root between 0 and 1 of the equation $x^3 - 6x + 4 = 0$.
29. Expand $f(x) = e^x \sin x$ as a Maclaurin series up to the term containing x^4 and compute the value of $\sin 34^\circ$.
30. Prove that an integer k in \mathbb{Z}_n is a generator of \mathbb{Z}_n if and only if $\gcd(k, n) = 1$.
31. Define $\phi: \mathbb{Z}_{12} \rightarrow \mathbb{Z}_{12}$ by $\phi(x) = 3x$. Find the kernel of ϕ .

(6 × 4 = 24)

Part D*Answer any two questions.**Each question carries 15 marks.*

32. (a) Find the Fourier sine series and cosine series of $f(x) = x^3$ ($0 < x < L$).
- (b) Find the Fourier series of $f(x) = \frac{x^2}{2}$ ($-\pi < x < \pi$). Deduce, $1 - \frac{1}{4} + \frac{1}{9} - \frac{1}{16} + \dots = \frac{\pi^2}{12}$.
33. (a) Solve $(y^2 + z^2)p - xyq = -zx$.
- (b) Find the integral surface of the $(x - y)y^2p + (y - x)x^2q = (x^2 + y^2)z$ through the curve $xz = a^3$, $y = 0$.
34. (a) Using Newton-Raphson method, find correct to four decimals the root of the equation $\sin x = 1 + x^3$ lies between -2 and -1.
- (b) Find the root of the equation $x^3 - 2x - 5 = 0$ by Regula-Falsi method, when it is given that the root lies between 2 and 3.
35. (a) Show that the set of all complex numbers with usual addition and multiplication forms a field. Is it a vector space over \mathbb{C} ? Justify.
- (b) Obtain the multiplication table for the cyclic subgroup S_5 generated by the permutation $\sigma = (1\ 2\ 4)(3\ 5)$. Is this group isomorphic to S_3 .

(2 × 15 = 30)