E 2500 (Pages : 3) Reg. No.................................

Name.....................................

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, NOVEMBER 2011

First Semester

DIFFERENTIAL CALCULUS AND TRIGONOMETRY

(Complementary Course for Physics/Chemistry/Petrochemicals/Geology/Food Science and Quality Control and Computer Maintenance and Electronics)

Time : Three Hours Maximum weight : 25

Part A (Objective Type Questions)

Answer **all** *questions.*
A bunch of **four** *questions has weight* 1.

I. 1 Find $\lim_{x \to \infty} \frac{(x+1)(2x+3)}{(x+2)(3x+4)}$.

2 State the Sandwich theorem.

3 If $y = \log(\log x)$ find $\frac{dy}{dx}$.

4 Differentiate :

$e^x \cos(5x+3)$ w.r.to x.

II. 5 Find the equation of the tangent line at (6, 4) on the graph of the function $f(x) = \frac{8}{\sqrt{x-2}}$.

6 Find the slope of the y-axis.

7 State the first derivative theorem for the local extreme values.

8 Verify Rolle's theorem for the function $f(x) = \frac{x^3}{3} - 3x$ in $[-3,3]$.

III. 9 Find the critical points of $f(x) = x^3 - 12x + 4$.

10 In the Mean Value theorem $f(a+h) = f(a) + hf^1(a+\theta h)$, show that $\theta = ½$ if (x) is a qudratic expression.

11 Define the partial derivative of $z = f(x, y)$ w.r. to x.

12 Write down the two dimensional Laplace's equation.

IV. 13 State the chain rule for the function of two independent variables.

Turn over

14 Find $\frac{\partial^2 f}{\partial x \partial y}$ if $f(x,y) = e^{xy}$.

15 Separate into real and imaginary parts the expression $\sin(\alpha + i\beta)$

16 Define the hyperbolic tangent of y.

$(4 \times 1 = 4)$

Part B (Short Answer Questions)

Answer any **five** *questions.*
Each question has weight 1.

17 Prove that

$$\lim_{x \to 0} \frac{a^x - 1}{x} = \log_e a.$$

18 Differentiate $\log\left(x^2 e^{mx}\right)$ w.r. to x.

19 Give the geometrical meaning of the Rolle's theorem.

20 Find the function $f(x)$ whose derivative is $\sin x$ and whose graph passes through the point (0, 2).

21 If z is a function of x and y and $x = e^u + e^{-v}$, $y = e^{-u} - e^{-v}$ then prove that

$$\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y}.$$

22 Show that $f(x,y) = \log\sqrt{x^2 + y^2}$ satisfies the Laplace's equation.

23 Use De Moivre's theorem to find the expansion of $\cos n\theta$ in terms of the trigonometrical functions of θ.

24 Prove that $\sin(x+y) = \sin x \cos y + \cos x \sin y$.

$(5 \times 1 = 5)$

Part C (Short Essay Questions)

Answer any **four** *questions.*
Each question has weight 2.

25 If $\sqrt{y+x} + \sqrt{y-x} = c$, show that $\frac{dy}{dx} = \frac{y}{x} - \sqrt{\frac{y^2}{x^2} - 1}$.

26 Differentiate $\log\left(xe^x\right)$ w.r. to $x \log x$.

27 State and prove the Mean Value theorem.

28 Find the critical points of

$$f(x) = x^{1/3}(x-4).$$

Identify the intervals on which f is increasing and decreasing. Also find the absolute extreme values.

29 Find $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$ in terms of r and s if

$$w = x + 2y + z^2, x = \frac{r}{s}$$
$$y = r^2 + \log s, z = 2r.$$

30 Separate into real and imaginary parts the quantity $\sin^{-1}(\cos\theta + i\sin\theta)$ where θ is real.

(4 × 2 = 8)

Part D (Essay Questions)

Answer any **two** *questions.*
Each question has weight 4.

31 (i) Prove that $\lim_{x \to a} \frac{x^n - a^n}{x - a} = n\, a^{n-1}$.

(ii) If $y = \left(x + \sqrt{1+x^2}\right)^m$ show that $\left(1+x^2\right)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - m^2 y = 0$.

32 (i) If $Z = f(x + ay) + \phi(x - ay)$, prove that $\frac{\partial^2 z}{\partial y^2} = a^2 \frac{\partial^2 z}{\partial x^2}$.

(ii) If $Z = \frac{\sin u}{\cos v}$ where $u = \frac{\cos y}{\sin x}$ and $v = \frac{\cos x}{\sin y}$, find $\frac{\partial z}{\partial x}$.

33 Sum to infinity the series $1 + c\cos\alpha + c^2\cos 2\alpha + \dots$

(2 × 4 = 8)