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Reg. No.....

Name.....

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, NOVEMBER 2013

First Semester

Complementary Course-DIFFERENTIAL CALCULUS AND TRIGONOMETRY

(Complementary Course for Physics/Chemistry/Petrochemicals/Geology Food Science and Quality Control/Computer Maintenance and Electronics)

[2013 Admissions]

Time: Three Hours

Maximum: 80 Marks

Part A

Short Answer Questions. (Answer all questions). Each question carries 1 mark.

- 1. Find $\lim_{x\to 1} \frac{x^2-1}{x-1}$.
- 2. Define the instantaneous rate of change of a function with respect to x.
- 3. State the extreme value theorem.
- 4. Define critical point of a function.
- 5. State Rolle's theorem.
- 6. What is the physical interpretation of the mean value theorem?
- 7. Define level curve of a function f(x, y).
- 8. Find $\frac{\partial f}{\partial y}$ if $f(x, y) = \sqrt{x^2 + y^2}$.
- 9. Define the hyperbolic sine of x.
- 10. What is the period of $\cosh(x+iy)$?

 $(10 \times 1 = 10)$

Part B

Brief Answer Questions. (Answer any eight questions).

Each question carries 2 marks.

- 11. If $3-x^3 \le f(x) \le 3 \sec x$ for all x, find $\lim_{x\to 0} g(x)$.
- 12. Find the slope of the curve $y = x + \frac{2}{x}$ at x = 1.

Turn over

- 13. If x = 2t + 3 and $y = t^2 + 1$, find the value of $\frac{dy}{dx}$ at t = 6.
- 14. Find the absolute minimum of $g(t) = 8t t^4$ on [-2, 1].
- 15. Show that the function $f(x) = x^4 + 3x + 1$ has exactly one zero in [-2, 1].
- 16. Given that the velocity v(t) = 9.8t + 5, s(0) = 10. Find the position s(t) of the body at time t.
- 17. Find $\frac{\partial f}{\partial x}$ and $\frac{\partial t}{\partial y}$ if $f(x, y) = x^2 y + \cos y + y \sin x$.
- 18. Use chain rule to find $\frac{dw}{dt}$ if $w = x^2 + y^2$, $x = \cos t$, $y = \sin t$. Also find $\frac{dw}{dt}$ at $t = \frac{\pi}{2}$.
- 19. Using implicit differentiation formula find $\frac{dy}{dx}$ if $y^2 x^2 \sin xy = 0$.
- 20. Define $\sin x$ and $\cos x$ in terms of exponential functions and show that $\sin (x+y) = \sin x \cos y + \cos x \sin y$.
- 21. Prove that $\cosh 3x = 4 \cosh^3 x 3 \cosh x$
- 22. Show that $\cosh^{-1} x = \log (x + \sqrt{x^2 1})$.

 $(8 \times 2 = 16)$

Part C

Short Essay Questions. (Answer any six questions).

Each question carries 4 marks.

- 23. Using the Sandwich Theorem find the horizontal asymptote of $y = 2 + \frac{\sin x}{x}$.
- 24. Find $\frac{d^2y}{dx^2}$ as a function of t if $x = t + \frac{1}{t}$, $y = t \frac{1}{t}$.
- 25. Use implicit differentiation to find $\frac{dy}{dx}$ and then find $\frac{d^2y}{dx^2}$ if $x^{\frac{3}{2}} + y^{\frac{3}{2}} = 1$.

- 26. If f'(x) = 0 at each point of an open interval (a, b), then prove that f(x) is constant for all x in (a, b).
- 27. Find the critical points of $f(x) = x^3 12x 5$ and identity the intervals on which f is increasing and decreasing.
- 28. Express $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$ in terms of r and s if:

$$w = x^2 + y^2$$
, $x = r - s$, $y = r + s$.

- 29. If resistors of R_1 , R_2 and R_3 ohms are connected in parallel to make an R-ohm resistor, the value of R can be found from $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$. Find the value of $\frac{\partial R}{\partial R_2}$ when $R_1 = 30$, $R_2 = 45$ and $R_3 = 90$ ohms.
- 30. Separate into real and imaginary parts of $\tan (\alpha + i\beta)$.
- 31. Express $\frac{\sin 6 \theta}{\sin \theta}$ in terms of $\cos \theta$.

 $(6 \times 4 = 24)$

Part D

Essay Questions. (Answer any two questions). Each question carries 15 marks.

- 32. (a) Let $f(x) = \sqrt{x-1}$ and $\varepsilon = 1$. Find a $\delta > 0$ such that for all x with $0 < |x-5| < \delta$ the inequality $|f(x)-2| < \varepsilon$ holds.
 - (b) Does the curve $y = x^4 2x^2 + 2$ have any horizontal tangents? If so where?
 - (c) Find the equation of the tangent at $t = \frac{\pi}{4}$ to the curve whose parametric equations are $x = \cos t$, $y = \sin t$, $0 \le t \le 2\pi$.
- 33. (a) State and prove the mean value theorem.
 - (b) For what values of a, m and b does the function:

$$f(x) = \begin{cases} 3, & x = 0 \\ -x^2 + 3x + a, & 0 < x < 1 \\ mx + b, & 1 \le x \le 2 \end{cases}$$

Satisfy the hypothesis of the mean value theorem on the interval [0, 2]?

Turn over

- (c) Find the critical points of the function f whose derivative f'(x) = x(x-1). Identity the intervals on which f is increasing and decreasing. Also find the local extreme values.
- 34. (a) Verify that $f_{xy} = f_{yx}$ if $f(x, y) = \log (2x + 3y)$.
 - (b) Find $\frac{dw}{dt}$ at t=0 if

$$w = x^2 + y^2$$
, $x = \cos t + \sin t$, $y = \cos t - \sin t$.

(c) Find $\frac{\partial w}{\partial r}$ when r=1, s=-1 if

$$w = (x + y + z)^2$$
, $x = r - s$, $y = \cos(r + s)z = \sin(r + s)$.

- 35. (a) Expand $\sin^7 \theta$ in a series of sines of multiples of θ .
 - (b) If $\sin(A+iB) = x+iy$ prove that $\frac{x^2}{\cosh^2 B} + \frac{y^2}{\sinh^2 B} = 1$ and $\frac{x^2}{\sin^2 A} \frac{y^2}{\cos^2 A} = 1$.
 - (c) Sum to infinity the series $c \sin \alpha + \frac{c^2}{2} \sin 2\alpha + \frac{c^3}{3} \sin 3\alpha + \dots$

 $(2 \times 15 = 30)$