

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, OCTOBER 2014**Third Semester**

Complementary Course—OPERATIONS RESEARCH—QUEUEING THEORY

(For Model II B.Sc. Mathematics)

[2013 Admissions]

Time : Three Hours

Maximum : 80 Marks

Part A*Answer all questions from this part.**Each question carries 1 mark.*

1. Define a saddle point.
2. Define the term competitive game.
3. Define the term pay-off matrix.
4. What is a float ?
5. Define critical path.
6. Define the term earliest time.
7. Define the term burst event.
8. Define a Poisson distribution.
9. What is queue length ?
10. What do you mean by idle period ?

(10 × 1 = 10)

Part B*Answer any eight questions.**Each question carries 2 marks.*

11. What are the assumptions made in the theory of games.
12. Find the value of the game.

		Player B	
		B ₁	B ₂
Player A	A ₁	2	6
	A ₂	-2	λ

13. Examine the matrix $\begin{bmatrix} 1 & 3 \\ -2 & 10 \end{bmatrix}$

for saddle points.

Turn over

14. Explain the terms latest time and total activity time.
15. What are the advantages of using Beta distribution in PERT analysis ?
16. Explain the term optimistic time estimate used in PERT.
17. What do you mean by 'Optimum project schedule'.
18. What is meant by the term critical activities ?
19. Explain the terms looping and dangling in network diagrams.
20. What do you understand by queue discipline ?
21. What are the essential features of queuing system ?
22. What do you understand by queue input ?

(8 × 2 = 16)

Part C

*Answer any six questions.
Each question carries 4 marks.*

23. Let $f(X, Y)$ be, such that both $\max_X \min_Y f(X, Y)$ and $\min_Y \max_X f(X, Y)$ exist. Prove that

$$\max_X \min_Y f(X, Y) \leq \min_Y \max_X f(X, Y).$$
24. Solve the game with the pay-off matrix $\begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix}$.
25. Solve graphically the game with pay-off matrix : $\begin{bmatrix} 1 & -1 & 2 \\ 2 & 3 & 1 \end{bmatrix}$.
26. Highlight the difficulties encountered in using network techniques.
27. Compare and contrast CPM and PERT.
28. Explain the terms activity variance and project variance in the context for project management.
29. Draw the network, given the following precedence relationships.

Event numbers :	1	2,3	4	5	6	7
Preceded by :	-	1	2,3	3	4,5	5,6
30. A television repairman finds that the time spent on his jobs has an exponential distribution with mean 30 minutes. If he repairs sets in the order in which they came in, and if the arrival of sets follows a Poisson distribution approximately with an average rate of 10 per 8-Hour day, what is the repairman's expected idle time each day ? How many jobs are ahead of the average set just brought in ?

31. For the queuing model (M/M/1 = N/FCFS), the steady state probability P_n is given by

$$P_n = \frac{(1-P)}{1-P^{N+1}} P^n, \quad 0 \leq n \leq N.$$

- (a) Obtain expressions for P_0 .
 (b) Obtain expected number of customers in the queue and system separately.

(6 × 4 = 24)

Part D

*Answer any two questions.
 Each question carries 15 marks.*

32. (a) Use the notion of dominance to simplify the following pay-off matrix and then solve the game :

$$\begin{bmatrix} 0 & 5 & -4 \\ 3 & 9 & -6 \\ 3 & -1 & 2 \end{bmatrix}$$

- (b) Write both the primal and the dual LP problems corresponding to the rectangular game with the following pay-off matrix :

$$\begin{bmatrix} 0 & 1 & 2 \\ 2 & 0 & 1 \\ 1 & 2 & 0 \end{bmatrix}$$

33. Discuss various steps involved in the applications of PERT and CPM.
 34. For a small project of 12 activities, the details are given below. Draw the network and find earliest occurrence time, latest occurrences time, critical activities and project completion time.

Activity	:	A	B	C	D	E	F	G	H	I	J	K	L
Dependence	:	-	-	-	B,C	A	C	E	E	D,F,H	E	I,J	G
Duration (days)	:	9	4	7	8	7	5	10	8	6	9	10	2

35. At a railway station, only one train is handled at one time. The railway yard is sufficient only for two trains to wait while the other is given signal to leave the station. Trains arrive at the station at an average rate of 6 per hour and the railway station can handle them on an average of 12 per hour. Assuming Poisson arrival and exponential service distribution, find the steady-state probabilities for the various number of trains in the system. Also find the average waiting time of a new train coming into the yard.

(2 × 15 = 30)