E 1218 (Pages : 3) Reg. No..............................

Name..................................

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, OCTOBER 2015

Third Semester

Core Course 3—CALCULUS

[Common for Model I, Model II Mathematics and B.Sc. Computer Applications]

(2013 Admission onwards)

Time : Three Hours Maximum : 80 Marks

Part A

Answer **all** *questions from this part.*
1 mark each.

1. State Leibnitz's theorem for the nth derivative of the product of two functions.
2. Define the envelope of a one parameter family of curves.
3. Write the Cartesian formula for the radius of curvature of a curve at a point on the curve.
4. State Euler's theorem on Homogeneous functions.
5. If $z = x^y$, find $\frac{\partial z}{\partial y}$.
6. Write the shell formula for revolution about y-axis to find the volume of a solid generated by revolving the region between the x-axis and the graph of $y = f(x) \geq 0, 0 \leq a \leq x \leq b$.
7. Write the surface area formula for revolution about the x-axis, a smooth curve $y = f(x)$ on $[a, b]$.
8. State Pappus's theorem for volumes.
9. Find the area of the region R bounded by $y = x$ and $y = x^2$ in the first quadrant.
10. Change the Cartesian integral $\int_{-1}^{1}\int_{0}^{\sqrt{1-x^2}} dydx$ into polar integral.

(10 × 1 = 10)

Part B

Answer any **eight** *questions.*
Each question carries 2 marks.

11. If $ax^2 + 2hxy + by^2 = 1$, show that $\frac{d^2y}{dx^2} = \frac{(h^2 - ab)}{(hx + by)^3}$.
12. Show that $e^{-x} < 1 - x + \frac{x^2}{2}$.
13. Find the radius of curvature of $y^2 = 4x$ at the point (x, y).

Turn over

14. If $u = a \cdot \tan^{-1}\frac{y}{x}$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$.

15. Find $\frac{du}{dt}$ where $u = \sin\frac{x}{y}, x = e^t, y = t^2$.

16. Fine the envelope of the family of straight lines $y = mx + a/m$, m being the parameter.

17. Find the area of the region bounded above *by* $y = x + 6$, bounded below *by* $y = x^2$, and bounded on the sides by the lines $x = 0$ and $x = 2$.

18. Find the volume of the solid generated when the region enclosed by $y = \sqrt{x}$, $y = 2$ and $x = 0$ is revolved about the y-axis.

19. Find the length of the curve $y = x^{3/2}$ from $x = 0$ to $x = 4$.

20. Evaluate $\int_{\pi}^{2\pi}\int_0^{\pi}(\sin x + \cos y)\,dxdy$.

21. Find the area enclosed by the lemniscate $r^2 = 4\cos 2A$.

22. Evaluate $\int_0^1\int_0^{1-y}\int_0^2 dxdzdy$.

(8 × 2 = 16)

Part C

Answer any **six** *questions.*
Each question carries 4 marks.

23. Find the nth derivative of $x^3 \log x$.

24. Using Maclaurin's series, prove that $\tan^{-1}x = x - \frac{x^3}{3} + \frac{x^5}{5}$.........

25. Find the points of inflexion on the curves, $y = \frac{x^3}{a^2 + x^2}$.

26. Examine $f(x, y) = x^2 - 3xy + y^2 + 2x$ for maxima and minima.

27. If $v = \frac{1}{r}$ and $r^2 = x^2 + y^2$, prove that $\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = \frac{1}{r^3}$.

28. Use the shell method to find the volume of the solid generated by revolving the region bounded by $y = x^2, y = 2 - x, x = 0$, for $x \geq 0$ about the y-axis.

29. Find the area of the surface generated by revolving the curve $y = x^3, 0 \leq x \leq \frac{1}{2}$, about the x-axis.

30. Evaluate $\int_0^{\pi}\int_0^{\pi}\int_0^{2\sin\phi} \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$.

31. Evaluate $\int_0^{2\pi}\int_0^{1}\int_r^{\sqrt{2-r^2}} dz r dr d\theta$.

(6 × 4 = 24)

Part D

Answer any **two** *questions.*
Each question carries 15 *marks.*

32. (i) If $y = \dfrac{\sin^{-1} x}{\sqrt{1-x^2}}$ show that $\left(1-x^2\right) y_{n+1} - (2n+1) x y_n - n^2 y_{n-1} = 0$.

(ii) Find the minimum of $x^2 + y^2 + z^2$, when $x + y + z = 3a$.

33. (i) Prove that the asymptotes of $x^2 y^2 = c^2\left(x^2 + y^2\right)$ are the sides of a square.

(ii) Find the extema of $z = \sin^2 x + \sin^2 y$ subject ot the condition $y - x = \pi/4$.

34. (i) Find the volume of the solid generated by revolving the region bounded by $y = 2x$, $y = x$ and $x = 1$ about the x-axis.

(ii) Find the area of the surface generated by revolving the curve $y = 2\sqrt{x}, 1 \le x \le 2$, about the x-axis.

35. (i) Calculate $\iint\limits_{P} \dfrac{\sin x}{x} \, dA$.

(ii) Evaluate $\int_0^1\int_0^{1-x} \sqrt{x+y}\,(y-2x)^2 \, dy dx$.

(2 × 15 = 30)