E 6022 (Pages : 3) Reg. No..............................

Name..............................

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, OCTOBER 2013

Third Semester

Mathematics

Core Course—3—CALCULUS

(Common for Model I and Model II Mathematics and B.Sc. Computer Applications)

[2011 Admission onwards]

Time : Three Hours Maximum Weight : 25

Part A

Answer **all** *questions.*

Each bunch of **four** *questions carries a weight of* 1.

I. 1. If $y = A\cos nx + B\sin nx$, show that $\frac{d^2y}{dx^2} + R^2 y = 0$.

2. State Taylor's theorem.

3. Define evolute of a curve.

4. Find the envelope of $y = mx + \frac{a}{m}$.

II. 5. The evolute of a curve is the envelope of its ———.

6. If $z = x/y$, find $\frac{\partial z}{\partial y}$.

7. Find the critical points of $f(x, y) = x^3 + y^3 - 3x - 12y + 10$.

8. State Euler's theorem for homogeneous functions.

III. 9. Write the formula for the volume generated when the area bounded by a curve, the y-axis and abcissae $y = c$, $y = d$ revolves about the y-axis.

10. Write the formula for the length of a smooth curve $x = g(y), c \le y \le d$.

11. State Fubini's theorem second form.

12. Write the formula for the volume of a solid of revolution about x-axis.

Turn over

IV. 13. Write the equation in polar co-ordinates corresponding to $\iint_R f(x, y)\, dx\, dy$.

14. What are the co-ordinate conversion formulas from spherical to cylindrical co-ordinates.

15. If $x = r\cos\theta$, $y = r\sin\theta$. Find $J(r, \theta)$, the Jacobian.

16. Define the triple integral of a function $f(x, y, z)$ over a bounded region in space.

(4 × 1 = 4)

Part B

Answer any **five** *questions.*
Each question carries a weight of 1.

17. If $ax^2 + 2hxy + by^2 = 1$, prove that $\frac{d^2y}{dx^2} = \frac{h^2 - ab}{(hx + by)^2}$.

18. Expand $y = e^x$ using Taylor's theorem around $x = 1$.

19. Find the radius of curvature of $ay^2 = x^3$ at (a, a).

20. Verify that $\frac{\partial^2 u}{\partial x \partial y} \neq \frac{\partial^2 u}{\partial y \partial x}$ if $u = \log\frac{x^2 + y^2}{xy}$.

21. Find $\frac{du}{dt}$ where $u = \sin(xy^2)$, $x \log f$, $y = e^t$.

22. Find the area of the region enclosed by $y = 2x - x^2$, $y = -3$.

23. Find the volume of the solid generated by revolving the region bounded by $y = \sec x$, $y = 0$, $x = -\pi/4$ and $x = \pi/4$ about x-axis.

24. Evaluate $\iint_R e^{x^2+y^2}\, dy\, dx$ where R is the semicircular region bounded by the x-axis and the curve $y = \sqrt{1 - x^2}$.

(5 × 1 = 5)

Part C

(Short Essay)

Answer any **four** *questions.*

Each question carries a weight of 2.

25. If $u = \sin^{-1}\frac{x}{y} + \tan^{-1}\frac{y}{x}$. Show that $x \cdot \frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$.

26. Show that the envelope of the parabolas $ay^2 = a^2(x - a)$ where a is a parameter is the curve $27ay^2 = 4x^3$.

27. Find the shortest and longest distance from the point $(1, 2, -1)$ to the sphere $x^2 + y^2 + z^2 = 24$.

28. Find the area of the region $y = 2\sin x$ and $y = \sin 2x, 0 \le x \le \pi$.

29. Use shell method to find the volume of the solid generated by revolving the region bounded by $y = \frac{1}{x}, y = 0, x = \frac{1}{2}, x = 2$.

30. Evaluate $\int_0^1 \int_0^\pi \int_0^\pi y \sin 2\, dx\, dy\, dz$.

$(4 \times 2 = 8)$

Part D

(Essay Type)

Answer any **two** *questions.*

Each question carries a weight of 4.

31. (a) If $y = \sin(m \sin^{-1} x)$, prove that $(1 - x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2 - n^2)y_n = r$.

    (b) Using Maclaurin's series show that $\log(1 - x + x^2) = -x + \frac{1}{2}x^2 + \frac{2}{3}x^3 + \frac{1}{4}x^4 + \ldots\ldots\ldots$

32. Find the area of the region in the first quadrant that is bounded above by $y = \sqrt{x}$ and below by the x-axis and the line $y = x - 2$ by integrating with respect to y.

33. Find the moment of inertia of a right circular cone of base radius a and height h about its axis.

$(2 \times 4 = 8)$