E 1688 (Pages : 3) Reg. No.................................

Name...................................

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, MARCH 2015

Sixth Semester

Core Course—LINEAR ALGEBRA AND METRIC SPACES

(For B.Sc. Mathematics Model I and II)

Time : Three Hours Maximum Weight : 25

Part A

Answer **all** *questions.*
Each bunch of **4** *questions has weight* 1.

I. 1 What do you mean by zero subspace of a vector space ?

2 Define linear span of a set S.

3 When an infinite set of vectors is said to be linearly independent ?

4 What do you mean by dimension of a vector space ?

II. 5 Determine whether $W = \{(x, y, x, y) : x, y \in I\}$ is a subspace of R^4.

6 Define a linear transformation.

7 Define Kernel of a linear transformation.

8 Give an example of a linear transformation on a vector space.

III. 9 Let $F : R^2 \to R^2$ be the mapping defined by $F(x, y) = (x + 1, y + 2)$. Is F a linear transformation.

10 Let $T : R^3 \to R^3$ be the projection mapping into the xy plane, defined by $T(x, y, z) = (x, y, 0)$. Find the image of T.

11 Define a closed set in a metric space.

12 Define an open sphere in a metric space.

IV. 13 How will we define boundary of a set ?

14 State Cantor's intersection theorem.

15 Define a Cauchy sequence.

16 Give an example of a metric space which is complete.

$(4 \times 1 = 4)$

Turn over

Part B

Answer any **five** *questions.*
Each question has weight 1.

17 Show that the vectors (1, 1, 0), (0, –1, 1), (–1, 0, –1) of R^3 are linearly dependent.

18 Prove that a set of vectors which contains at least one zero vector is linearly dependent.

19 Let $T: V \to V$ be a linear map and $x_1, x_2 ... x_4 \in V$. If $T(X_1), T(X_2) ... T(X_4)$ are linearly independent vectors of V, then prove that $X_1, X_2 ... X_4$ are also linearly independent.

20 Let $T: R^2 \to R^2$ be a linear map having the following effect on the indicated vectors. T (1, 1) = (2, 1), T (–1, 1) = (6, 3). Compute T (1, 0) and T (0, 1).

21 Let X be a metric space. If $\{x\}$ is a subset of X consisting of a single point, show that its complement is open.

22 In any metric space show that the empty set and the full space are closed sets.

23 Show that closed subspace of a complete metric space is complete.

24 Explain the concept of continuity in a metric space.

(5 × 1 = 5)

Part C

Answer any **four** *questions.*
Each question has weight 2.

25 Determine the co-ordinate representation of the matrix $\begin{bmatrix} 4 & 3 \\ 6 & 2 \end{bmatrix}$ with respect to the basis

$$S = \left\{ \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \right\}.$$

26 Let V be the vector space of ordered pairs of complex numbers over the real field R. Show that the set $S = \{(1, 0), (i, 0), (0, 1) (0, i)\}$ is a basis of V.

27 Find the matrix of the following linear transformation in R^3 relative to the usual basis. $T(x, y, z) = (x, y, 0)$.

28 If $S = \{v_1, v_2 ... v_4\}$ is a basis for a vector space V. Prove that any set containing more than n vectors is linearly dependent.

29 Let X be a metric space. Show that a subset G of X is open $\Leftrightarrow$ it is a union of open spheres.

30 Show that a complete subspace of a complete metric space is closed.

(4 × 2 = 8)

Part D

Answer any **two** *questions.*
Each question has weight 4.

31 Prove that if $v_1, v_2 ... v_4$ is a basis for V and if $w_1, w_2 ... w_m$ in V are linearly independent, then $m \leq n$.

32 Prove that if B is obtained from A by an elementary row operation, then the row space of A is the same as the row space of B. Hence determine the row rank of $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 7 \\ 3 & 6 & 10 \end{bmatrix}$.

33 Let X and Y be metric spaces and f a mapping of X into Y. Show that f is continuous if and only if $f^{-1}(G)$ is open in X whenever G is open in Y.

(2 × 4 = 8)