E 7514 (Pages : 3) Reg. No.................................

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B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, MARCH 2014

Sixth Semester

Core Course—LINEAR ALGEBRA AND METRIC SPACES

(For B.Sc. Mathematics Model I and II)

Time : Three Hours Maximum Weight : 25

Part A

Answer **all** *questions.*
Each bunch of 4 questions has weight 1.

I. 1 Is the set $w = \{(x_1, x_2, 0) : x_1, x_2 \in R\}$ a subspace of R^3.

2 What do you mean by dimension of a vector space ?

3. Give a basis for the set of all 2×3 matrices over a field k.

4 In a set of vectors in a vector space, one element is zero. Is that set linearly independent.

II. 5 Whether the set of integers form a vector space over R.

6 Let $T : R^2 \to R^2$ defined by $T(a, b) = (3a, b)$. Is T a linear transformation.

7 Show that Kernel of a linear transformation is a subspace of the vector space.

8 What do you mean by column rank of a matrix ?

III. 9 Let $T : R^3 \to R^3$ be the projection mapping into the xy-plane defined by $T(x, y, z) = (x, y, 0)$. Find the Kernel of T.

10 A Linear transformation $T : U \to V$ is one-one if and only if kernel of T = ———.

11 Define a metric space.

12 Define an open set in a metric space.

IV. 13 How will we define the interview of a set in a metric space ?

14 Define closure of a set.

15 State Cantor's intersection theorem.

16 Define a complete metric space.

$(4 \times 1 = 4)$

Turn over

Part B

Answer any **five** *questions.*
Each question has weight 1.

17 Find the non-trivial linear relations satisfied by the vectors $(2, 1, 1), (3, -4, 6), (4, -9, 11)$.

18 Determine whether the following vectors in R^3 linearly independent $\{(1,0,1), (1,1,1), (0,0,1)\}$.

19 Let $B = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and let T be the linear operator on R^2 defined by $T(X) = XB$ where

$X = (x, y) \in R^2$.

Find the matrix of T relative to the basis $\{(1, 01, (0, 1)\}$ of R^2.

20 Show that the mapping $F : R^3 \to R$ defined by $T(x, y, z) = 2x - 3y + 4z$ is linear.

21 In a metric space X, show that any intersection of closed sets is closed.

22 In a metric space show that each open sphere is an open set.

23 Let X and Y be metric spaces and f a mapping of X into Y. If f is a constant map, show that f is continuous.

24 Define a nowhere dense set. Give an example of a nowhere dense set.

$(5 \times 1 = 5)$

Part C

Answer any **four** *questions.*
Each question has weight 2.

25 Define a vector space. Illustrate if by an example.

26 Let V be the vector space of 2×3 matrices over R. Show that the matrices $A = \begin{bmatrix} 2 & 1 & -1 \\ 3 & -2 & 4 \end{bmatrix}$,

$B = \begin{bmatrix} 1 & 1 & -3 \\ -2 & 0 & 5 \end{bmatrix}$, $C = \begin{bmatrix} 4 & -1 & 2 \\ 1 & -2 & 3 \end{bmatrix}$ are linearly independent.

27 Prove that the image of a linear transformation $T : U \to V$ is a subspace of V.

28 Let T be the linear transformation defined by $T_1(x, y) = (2x - 3y, x + y)$ or R^2. Find the matrix of T relative to the basis $\{(1, 2), (2, 3)\}$.

29 Let X be a metric space with metric d. Show that d_1 defined by $d_1(x, y) = \frac{d(x, y)}{1 + d(x, y)}$ is also a metric on X.

30 Let X and Y be metric spaces and f be a mapping of X into Y. Show that f is continuous at x_0 if and only if $x_n \to x_0 \Rightarrow f(x_n) \to f(x_0)$.

(4 × 2 = 8)

Part D

Answer any **two** *questions.*
Each question has weight 4.

31 If $v_1, v_2, \ldots v_n$ in V have W as linear span and if $v_1, v_2, \ldots v_k$ are linearly independent, the show that we can find a subset of $v_1, v_2, \ldots v_n$ of the form $v_1, v_2, \ldots v_k, v_{i_1}, v_{i_2}, \ldots v_{i_r}$ consisting of linearly independent elements whose span is also W.

32 Let T be a linear transformation from an n dimensional vector space V into W and let $\{v_1, v_2, \ldots v_k\}$ be a basis for the kernel of T. If this basis is extended to a basis $\{v_1, v_2, \ldots v_k, v_{k+1}, \ldots v_n\}$ for V, then prove that $\{T(v_{k+1}), T(v_{k+2}) \ldots T(v_n)\}$ is a basis for the image T.

33 If $\{A_n\}$ is a sequence of nowhere dense sets in a complete metric space X, then show that there exists a point of X which is not in any of the A_n's.

(2 × 4 = 8)