E 3406 (Pages : 3) Reg. No.................................

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B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, MARCH/APRIL 2012

Sixth Semester

Core Course—LINEAR ALGEBRA AND METRIC SPACES

(For Model—I And Model—II B.Sc. Mathematics)

Time : Three Hours Maximum Weight : 25

Part A

Answer **all** *questions.*
A bunch of **four** *questions has weight* 1.

I. 1 State whether true *or* false : The set {[*a*, *b*]} of all real two dimensional row matrices with stand and matrix addition and scalar multiplication defined by $\alpha \cdot [a\, b] = [0\, 0]$ is a vector space.

2 Give an example of a subspace of $\mathbb{R}^2$ that has only one element.

3 Give a Geometrical interpretation for two vectors in $\mathbb{R}^2$ to be linearly dependent.

4 What is the dimension of the vector space $\mathbb{M}_{2 \times 3}$.

II. 5 Define row space of a matrix

6 Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be defined by $T[a\, b] = [a\, 0]$ Then $T^2\, [a, b] = \ldots$ for all a, b *in* $\mathbb{R}$.

7 If $T : \mathbb{R}^2 \to \mathbb{R}^3$ is a linear transformation then $T\, [0, 0] = \ldots$

8 Define Kernel of a linear transformation.

III. 9 If the linear transformation $T : V \to W$ is onto then Im (T) = . . .

10 If $T : \mathbb{R}^4 \to \mathbb{R}^3$ is a linear transformation with rank (T) = 2, then nullity (T) = ...

11 Define the diameter *d* (A) of a nonempty subset A of a metric space (X, *d*).

12 Consider the real line $\mathbb{R}$ with usual metric Give an example of an open set in $\mathbb{R}$ which is not an open interval.

IV. 13 What is a subset F of a metric space X said to be closed ?

14 Define the boundary of a subset A of a metric space.

Turn over

15 State Cantors intersection theorem.

16 State a characterization of a continuous mapping of a metric space X to a metric space Y in terms of open sets.

$(4 \times 1 = 4)$

Part B

Answer any **five** *questions.*
Each question has weight 1.

17 Determine whether $S = p(t) \in P^2 : p(2) = 0\}$ is a subspace of $\mathcal{P}^2$

18 Determine whether $S = \left\{\begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix}\right\}$ is a basis for $\mathbb{R}^2$, considered as column matrices.

19 Define a linear transformation. If V and W are vector spaces show that $T : V \to W$ defined by $T(v) = 0$ for all v in V is a linear transformation.

20 Find the Kernel and nullity of the linear transformation $T : \mathcal{P}^2 \to M_{2 \times 2}$ defined by $T(at^2 + bt + c) = \begin{bmatrix} a & 2b \\ 0 & a \end{bmatrix}$ for all a, b, c in $\mathbb{R}$.

21 In any metric space X, prove that each open sphere is an open set.

22 Define the closure $\overline{A}$ of a subset A of a metric space and show that A is closed if and only if $A = \overline{A}$.

23 Show that every convergent sequence in a metric space in a Cauchy sequence.

24 Let X, Y be metric spaces. If the mapping $f : X \to Y$ is uniformly continuous then prove that f is continuous.

$(5 \times 1 = 5)$

Part C

Answer any **four** *questions.*
Each question has weight 2.

25 Prove that the span of a set of vectors $S = \{V_1, V_2, \ldots, V_n\}$ in a vector space V is a subspace of V.

26 Determine the dimension of $\mathcal{P}^n$.

27 Let $T:\mathbb{R}^2 \to \mathbb{R}^3$ be defined by $T\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} a+b \\ a-b \\ 2b \end{bmatrix}$. Find the matrix representation of T with respect to the bases $B = \left\{\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix}\right\}$ or $\mathbb{R}^2$ and $C = \left\{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\right\}$ for $\mathbb{R}^3$.

28 Determine the Kernel of the matrix $A = \begin{bmatrix} 1 & 1 & 5 \\ 2 & -1 & 1 \end{bmatrix}$.

29 Let X be a metric space. Prove that a subset F of X is closed if and only if its complement F' is open.

30 Let X be a complete metric space and Y be a subspace of X. If Y is closed then prove that Y is complete.

(4 × 2 = 8)

Part D

Answer any **two** *questions.*
Each question has weight 4.

31 If $S = \{v_1, v_2, \ldots, v_n\}$ is a basis for a vector space V, then prove that any set containing more than n vectors is linearly dependent. Further, deduce that every basis for a finite dimensional vector space must contain the same number of vectors.

32 Let T be a linear transformation from an n-dimensional vector space V into W. Prove that rank (T) + nullity (T) = dim (V).

33 Prove that every non-empty open set on the real line is the union of a countable class of open intervals.

(2 × 4 = 8)