E 5863 (Pages : 2) Reg. No...............................

Name...............................

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, MARCH 2017

Sixth Semester

Core Course—LINEAR ALGEBRA AND METRIC SPACES

(2013 Admission onwards)

Time : Three Hours Maximum : 80 Marks

Part A

Answer **all** *questions each in* **a sentence** *or* **two.**
Each question carries 1 mark.

1. Define additive inverse of any vector in a vector space.
2. Define subspace of a vector space.
3. Give an example of a linearly dependent set in $\mathbb{R}^2$.
4. Give an example of an one to one function.
5. Define rank of a linear transformation.
6. Define the linear transformation 'rotation'.
7. Define metric space.
8. Show that full space is an open set in any metric space.
9. Define closed sphere.
10. Define complete metric space.

(10 × 1 = 10)

Part B (Short Notes)

Answer any **eight** *questions.*
Each question carries 2 marks.

11. Show that the zero vector in a vector space V is unique.
12. Check whether the set of all 2 × 2 real matrices $A = [a_{ij}]$ with $a_{11} = -a_{22}$ under standard matrix addition and scalar multiplication is a vector space.
13. For any vector space V, show that the subset containing only the zero vector is a subspace.
14. Prove or disprove that the function $T : \mathbb{R}^2 \to \mathbb{R}^2$ given by $T[a, b] = [a^2, b^2]$ is linear.
15. If $T : V \to W$ is a linear transformation, then prove that $T(0) = 0$.
16. If a linear transformation $T : V \to W$ is one to one then prove that the image of every linearly independent set of vectors in V is a linearly independent set of vectors in W.

Turn over

17. In any metric space X, show that the empty set 0 and the full set X are closed sets.

18. Let X be a metric space. Then prove that any intersections of closed sets in X is closed.

19. Let X be an arbitrary metric space, and let A be a subset of X. Prove that $A = \bar{A}$ if A is closed.

20. Define nowhere dense set and give an example.

21. Define convergence of a sequence and give an example.

22. Define boundary of a set and give an example.

(8 × 2 = 16)

Part C

Answer any **six** *questions.*
Each question carries 4 marks.

23. Show that a set of vectors in a vector space V that contains the zero vector is linearly dependent.

24. Determine whether the set of all real valued continuous functions on the interval [0, 1] under standard function addition and scalar multiplication is a vector space.

25. find a basis for the span of the vectors in $S = \{t, t+1, t-1, 1\}$.

26. Prove that a matrix A is similar to a matrix B and B is similar to another matrix C then A is similar to C.

27. Find a co-ordinate representation for the vector $v = 4t^2 + 3t + 2$ in $\mathbb{P}^2$ with respect to the basis $\mathbb{C} = \{t^2 + t, t+1, t-1\}$.

28. Show that the kernal of a linear transformation is a subspace of the domain.

29. In any metric space, show that each open sphere is an open set.

30. Define Cantor set and explain its construction.

31. Let X be a complete metric space, and let Y be a subspace of X. Then show that if Y is closed then it is complete.

(6 × 4 = 24)

Part D (Essays)

Answer any **two** *questions.*
Each question carries 15 marks.

32. Show that a finite set of vectors is linearly dependent if and only if one of the vectors is a linear combination of the vectors that precede it, in the ordering established by listing of vectors in the set.

33. Define kernel of a linear transformation. Show that a linear transformation $T : V \to W$ is one-to-one if and only if the kernal of T contains just the zero vector.

34. Show that every non-empty open set in the real line is the union of a countable disjoint class of open intervals.

35. State and prove Cantor's intersection theorem.

(2 × 15 = 30)