E 1687 (Pages : 2) Reg. No.....

Name.....

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, MARCH 2015

Sixth Semester

Core Course—DISCRETE MATHEMATICS

(For B.Sc. Mathematics Model I and II)

Time : Three Hours Maximum Weight : 25

Part A

Answer **all** *questions.*

Each bunch of four questions has weight 1.

I. 1 Define graph isomorphism.

2 Give an example of a self complementary graph.

3 State first theorem of graph theory.

4 When can you say that a graph is connected ?

II. 5 Define incidence matrix of a graph G.

6 Draw a tree with *six* vertices.

7 Give an example of a simple graph G such that L (G) is Euler but G is not.

8 Define maximal non-Hamiltonian graph.

III. 9 How many different Hamiltonian cycles does $K_{n,n}$ have ?

10 Define perfect matching.

11 State Hall's marriage theorem.

12 Define "running key".

IV. 13 Is the sequence 7, 27, 47, 97, 197, 397 superincreasing ?

14 State the duality principle on posets.

15 Define complete lattice.

16 When can you say that the interval $[a, b]$ is complemented ?

$(4 \times 1 = 4)$

Part B

Answer any **five** *questions.*

Each question has weight **1.**

17 Let G be a graph in which there is no pair of adjacent edges. What can you say about the degree of the vertices in G ?

Turn over

18 Prove that the joint of two vertex disjoint complete graphs is a complete graph.

19 Prove that a vertex V of Q tree T is a cut vertex if and only if $d(V) > 1$.

20 Which of the wheel graphs W_n's are Euler ?

21 Write a note on Chinese postman problem.

22 Using the linear cipher $C \equiv 5P + 11 (\text{mod } 26)$, encrypt the message "NUMBER THEORY IS EASY".

23 Show that union of two sublattices may not be a sublattice.

24 Show that a lattice L is a chain if and only if every non-empty subset of it is a lattice.

$(5 \times 1 = 5)$

Part C

Answer any **four** *questions.*
Each question has weight 2.

25 Given any two vertices u and v of a graph G, prove that every u–v walk contains a u–v path.

26 Prove that an edge e of a graph G is a bridge if and only if e is not a part of any cycle in G.

27 Prove that a tree has atmost one perfect matching.

28 Let G be a graph in which the degree of every vertex is at least two, then prove that G contains a cycle.

29 Find the unique solution of the superincreasing knapsack problem :

$51 = 3x_1 + 5x_2 + 9x_3 + 18x_4 + 37x_5$.

30 Prove that dual of a lattice is a lattice.

$(4 \times 2 = 8)$

Part D

Answer any **two** *questions.*
Each question has weight 4.

31 Define 2-connected graph. State and prove Whitney's theorem on 2-connected graphs.

32 Define maximum matching and M-augmenting path. Prove that a matching M in a graph G is a maximum matching if and only if G contains no M-augmenting path.

33 Prove that any non-modular lattice L contains a sublattice isomorphic with the pentagonal lattice.

$(2 \times 4 = 8)$