E 5862 (Pages : 3) Reg. No.....

Name.....

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, MARCH 2017

Sixth Semester

Core Course—DISCRETE MATHEMATICS

(For B.Sc. Mathematics Model I and Model II)

[2013 Admission onwards]

Time : Three Hours Maximum : 80 Marks

Part A (Short Answer Questions)

Answer all *questions.*
Each question carries 1 *mark.*

1. Give the number of vertices and the number of edges in the complete bipartite graph $K_{m,n}$.
2. Define incidence matrix of a graph.
3. Define a bridge.
4. Find the vertex connectivity of the complete graph K_n.
5. Define a Hamilton cycle.
6. State Hall's marriage theorem.
7. What is meant by a monoalphabetic cipher ?
8. State Knapsack problem.
9. Write the dual of $a \vee (b \wedge c) \leq b \wedge (a \vee c)$ for all a, b, c in L with $a \leq b$.
10. Define a complemented lattice.

$(10 \times 1 = 10)$

Part B (Brief Answer Questions)

Answer any eight *questions.*
Each question carries 2 *marks.*

11. Let G be a graph with n vertices, t of which have degree k and others of degree $k + 1$. Prove that $t = (k + 1)n - 2e$, where e is the number of edges in G.
12. Define underlying simple graph of a graph and illustrate with an example.
13. Define walk in a graph. When is a walk said to be a path ?
14. Let G be a graph with no loops. If for every pair of distinct vertices u and v of G there is precisely one path from u to v, then prove that G is a tree.
15. Define Spanning tree of a graph. Draw two nonisomorphic spanning trees of K_4.

Turn over

16. Interpret the Chinese Postman Problem in graph theoretical terms.
17. Define closure of a graph.
18. Define an M-augmenting path and give an example.
19. Describe Caesar cipher using congruence theory.
20. Define a superincreasing sequence and give an example.
21. Define isomorphic lattices. Give an example.
22. Prove that every finite lattice is bounded.

(8 × 2 = 16)

Part C (Short Essay Type Questions)

Answer any **six** *questions.*
Each question carries 4 *marks.*

23. State and prove the first theorem of graph theory. Deduce that in any graph G there is an even number of odd vertices.
24. If T is a tree with n vertices, prove that it has precisely $n - 1$ edges.
25. Prove that every connected graph has a spanning tree.
26. Let G be a graph in which the degree of every vertex is at least two. Prove that G contains a cycle.
27. Prove that every k-regular graph with $k > 0$ has a perfect matching.
28. Encrypt the message NUMBER THEORY using the cipher $C = 5P + 11 \pmod{26}$.
29. Find the unique solution of the superincreasing Knapsack problem :

$$118 = 4x_1 + 5x_2 + 10x_3 + 20x_4 + 41x_5 + 99x_8.$$

30. Prove that the dual of a lattice is a lattice.
31. Prove that a sublattice S of a lattice L is a convex sublattice if and only if for all a, b in S, $(a \leq b)$, $[a, b] \subseteq S$.

(6 × 4 = 24)

Part D (Essay Questions)

Answer any **two** *questions.*
Each question carries 15 *marks.*

32. (a) Let G be a non-empty graph with at least two vertices. If G has no odd cycles, then prove that G is bipartite.

    (b) Let G be a graph with n vertices, where $n \geq 2$. Prove that G has at least two vertices that are not cut vertices.

33. (a) Prove that a connected graph is Euler if and only if the degree of every vertex is even.

    (b) Prove that a simple graph G is Hamiltonian if and only if closure of G is Hamiltonian.

34. (a) Use the Hill cipher :

$C_1 = 5P_1 + 2P_2 \pmod{26}$

$C_2 = 3P_1 + 4P_2^2 \pmod{26}$

to encipher the message GIVE THEM TIME.

(b) The cipher text ALXWU VADCOMO has been enciphered with the cipher

$C_1 \equiv 4P_1 + 11P_2 \pmod{26}$

$C_2 \equiv 3P_1 + 8P_2 \pmod{26}$

Derive the plaintext.

35. (a) Draw the diagram of the lattice of factors of 20 under divisibility and show that it is same as that of the product of two chains with three and two element.

(b) Define a distributive lattice and give an example.

(c) Prove that every distributive lattice is modular.

$(2 \times 15 = 30)$