**E 1686** (Pages : 3) Reg. No..............................

Name..................................

# B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, MARCH 2015

## Sixth Semester

Core Course—COMPLEX ANALYSIS

(For B.Sc. Mathematics Model I and II)

Time : Three Hours Maximum Weight : 25

### Part A

*Answer* **all** *questions.*
*Each bunch of 4 questions has weight* 1.

I. 1 Evaluate $\lim_{z \to i} \frac{i z^3 - 1}{z + i}$.

2 Define continuity of a function at a point.

3 Give an example of a function which is differentiable only at the origin.

4 Show that the function $u(x, y) = 2x(1 - y)$ is a harmonic function.

II. 5 Show that $\cos h\,(iz) = \cos z$.

6 Show that $f(z) = xy + iy$ is nowhere analytic.

7 Evaluate $\int_1^2 \left(\frac{1}{t} - i\right)^2 dt$.

8 If C is any simple closed contour then find the value of $\int_C \exp\left(z^3\right) dz$.

III. 9 If C is the unit circle $|z| = 1$, then what is the value of $\int_C \frac{z^2}{z-3}\, dz$.

10 If $z_0$ is any point interior to a positively oriented simple closed contour C, what is the value of

$\int_C \frac{dz}{z - z_0}$.

Turn over

11 If C is the arc of the circle $|z| = 2$. from $z = 2$ to $z = 2i$, show that $\left|\int_C \frac{dz}{z^2 - 1}\right| \leq \pi | 3$.

12 If C is any closed contour lying in the open disk $|z| < 2$, then what is $\int_C \frac{z e^z}{\left(z^2 + 9\right)^5} dz$.

IV. 13 Write the Maclaurin's series expansion of $e^z$.

14 Define the term pole of order $m$ for a function $f(z)$.

15 What is the nature of the singular point $z = 0$ of $e^{1/z}$.

16 Consider $f(z) = \frac{1}{z^2 (1 + z)}$. What is the residue of $f(z)$ at the origin.

(4 × 1 = 4)

## Part B

*Answer any* **five** *questions.*
*Each question has weight* 1.

17 Show that $\exp(z + \pi i) = -\exp z$.

18 Show that $|\sinh x| \leq |\cosh z| \leq \cosh x$.

19 Show that for the function $f(z) = z^2$, Cauchy Riemann equations are satisfied everywhere.

20 Evaluate $\int_C \frac{z + 2}{z} dz$, where C is the circle $z = 2e^{i\theta} (0 \leq \theta \leq 2\pi)$.

21 Evaluate $\int_C \tan z \, dz$, where C is the unit circle $|z| = 1$, in either direction.

22 State Laurent's theorem.

23 Show that $\int_C \exp\left(\frac{1}{z^2}\right) dz = 0$.

24 Find the order of the pole and the residue of the point $z = 0$ for the function $f(z) = \frac{\sinh z}{z^4}$.

(5 × 1 = 5)

## Part C

*Answer any* **four** *questions.*
*Each question has weight 2.*

25 Show that the function $u(x, y) = \frac{y}{x^2 + y^2}$ is harmonic in some domain and find a harmonic conjugate $v(x, y)$.

26 State and prove the sufficient conditions for differentiability.

27 State and prove Cauchy's integral formula.

28 State and prove Liouville's theorem.

29 Give two Laurent series expansion in powers of $z$ for the function $f(z) = \frac{1}{z^2(1-z)}$ and specify the regions in which the expansions and valid.

30 State and prove Cauchy's residue theorem.

(4 × 2 = 8)

## Part D

*Answer any* **two** *questions.*
*Each question has weight 4.*

31 Let $C_R$ denote the upper half of the circle $|z| = R$ $(R > 2)$, taken in the counterclockwise direction.

Show that $$\left| \int_{C_R} \frac{2z^2 - 1}{z^4 + 5z^2 + 4} \, dz \right| \leq \frac{\pi R \left(zR^2 + 1\right)}{\left(R^2 - 1\right)\left(R^2 - 4\right)}$$

Then show that the value of the integral tends to zero as $R \to \infty$.

32 State and prove Taylor's theorem.

33 Use residues to evaluate the definite integral $\int_0^{2\pi} \frac{d\theta}{5 + 4\sin\theta}$.

(2 × 4 = 8)