E 3405 (Pages : 3) Reg. No..............................

Name..............................

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, MARCH/APRIL 2012

Sixth Semester

Core Course—DISCRETE MATHEMATICS

(For Model I and Model II B.Sc. Mathematics)

Time : Three Hours Maximum Weight : 25

Part A

Answer **all** *questions.*

Each bunch of 4 questions has weight 1.

I. 1. Define a complete bipartite graph.

2. Define a vertex deleted subgraph of a graph.

3. Draw the complement of the graph $K_{2,2}$.

4. Define a trail in a graph.

II. 5. Define spanning tree of a graph.

6. If e is a bridge in a graph G, then $W(G-e)$ = ——— (Fill in the blank).

7. Define an Eulerian graph.

8. Draw a graph having a Hamilton path but not having a Hamilton cycle.

III. 9. Is K_4 Eulerian ? Why ?

10. Define a matching in a graph.

11. Define neighbour set of a set S of veritices of a graph.

12. Encrypt the message RETURN HOME using Caesar Cipher.

IV. 13. Give an example of a super increasing sequence with five terms.

14. Is the poset {2, 3, 4, 6} under divisibility a Lattice ? Why ?

15. Define a modular lattice.

16. When is an element of a lattice said to be join irreducible ?

(4 × 1 = 4)

Part B

Answer any **five** *questions.*
Each question has weight 1.

17. State and prove the first theorem of Graph theory.

18. Define a self-complementary graph. Give an example with justification.

Turn over

19. Prove that an edge e of a graph G is a bridge, then e is not a part of any cycle in G.
20. If a connected graph G is Eulerian, then prove that the degree of every vertex is even.
21. For which values $n \geq 2$, does K_n have a perfect matching ?
22. Using the linear cipher $C \equiv 5P + 11 \pmod{26}$ encrypt the message NUMBER THEORY.
23. Prove that a finite lattice has least and greatest elements.
24. Prove that a chain is a distributive lattice.

(5 × 1 = 5)

Part C

Answer any **four** *questions.*
Each question has weight 2.

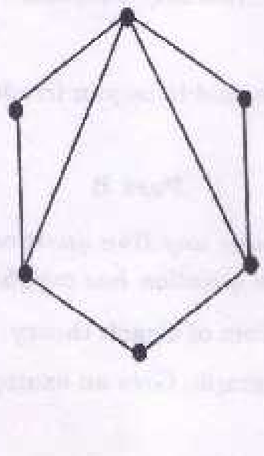
25. Let u and v be distinct vertices of a tree T. Prove that there is precisely one path from u to v.
26. Let G be a simple graph with at least three vertices. Prove that G is 2-connected if and only if for each pair of distinct vertices u and v of G, there are two internally disjoint $u-v$ paths in G.
27. Let G be a simple graph with n vertices where $n \geq 3$ and the degree $d(v) \geq \frac{n}{2}$ for every vertex v of G. Then prove that G is Hamiltonian.
28. Prove that a matching M in a graph G is a maximum matching if and only if G contains no M-augmenting path.
29. Explain how encryption and decryption are carried out in RSA cryptosystem.
30. Prove that the normal subgroups of a group ordered by inclusion form a modular lattice.

(4 × 2 = 8)

Part D

Answer any **two** *questions.*
Each question has weight 4.

31. Let G be a non-empty graph with at least two vertices. Prove that G is bipartite if and only if it has no odd cycles.
32. Define the closure C(G) of a simple graph G. Illustrate the construction of C(G) of the following graph G.



Further prove that a simple graph G is Hamiltonian if and only C(G) is Hamiltonian and deduce that if C(G) is complete, then G is Hamiltonian.

33. Define a complemented lattice. Is the pentagonal lattice complemented ? Justify your claim. Also show that the dual of a complemented lattice is complemented. Show further that if an element of a distributive lattice has a complement then that complement is unique.

$(2 \times 4 = 8)$