E 3404 (Pages : 3) Reg. No...............................

Name...............................

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, MARCH/APRIL 2012

Sixth Semester

Core Course—COMPLEX ANALYSIS

(For Model I and Model II B.Sc. Mathematics)

Time : Three Hours Maximum Weight : 25

Part A

Objective Type Questions.

Answer **all** *the questions. Each bunch of* **four** *questions has weight* 1.

I. 1. Write the domain of definition of the function $f(z) = \frac{z}{z + \bar{z}}$.

2. When a function f is said to be continuous at a point in its domain ?

3. Define an entire function.

4. When z_0 is called a singular point of a function f ?

II. 5. Show that $u(x, y) = x^2 - y^2$ is harmonic.

6. Show that $\exp(z + \pi i) = -\exp z$.

7. Define the hyperbolic cosine of a complex variable z.

8. Evaluate $\int_0^{\pi/6} e^{it} dt$.

III. 9. When an arc C is said to be simple ?

10. State Cauchy–Goursat theorem.

11. State Liouville's theorem.

12. What is maximum modulus principle ?

IV. 13. When the series $\sum_{n=1}^{\infty} Z_n$ of complex numbers is said to be absolutely convergent ?

14. Find the residue at $z = 0$ of the function $\frac{z - \sin z}{z}$.

Turn over

15. Define an essential singular point of f.

16. Find the Cauchy principal value of $\int_{-\infty}^{\infty} x\, dx$.

(4 × 1 = 4)

Part B

Short answer questions.

Answer any **five** *questions. Each question has weight* 1.

17. Show that $f(z) = \bar{z}$ is no where differentiable.

18. Show that if e^{z} is real, then $\mathrm{Im}\, z = n\pi, n = 0, \pm 1, \pm 2, \ldots$

19. Find the principal value of $(-i)^{i}$.

20. Show that $\int_C \frac{z^2}{z-3} dz = 0$, where C is the unit circle in either direction.

21. Find $\int_C \frac{1}{z^2+4} dz$, where C is the circle $|z-i| = 2$ in the positive sense.

22. Show that the sequence $z_n = \frac{1}{n^3} + i, n = 1, 2, 3, \ldots$ converges to i.

23. Show that $z_0 = 0$ is a removable singular point of the function $f(z) = \frac{1-\cos z}{z^2}$.

24. State Jordan's Lemma.

(5 × 1 = 5)

Part C

Short essay questions.

Answer any **four** *questions. Each question has weight* 2.

25. Derive the Cauchy–Riemann equations.

26. If a function $f(z) = u(x, y) + iv(x, y)$ is analytic in a domain D, then prove that its component functions u and v are harmonic in D.

27. Evaluate $\int_C \frac{z+2}{z} dz$, where C is the semicircle $z = 2e^{i\theta}, 0 \leq \theta \leq \pi$.

28. State and prove the fundamental theorem of algebra.

29. Show that when $z \neq 0$, $\frac{e^z}{z^2} = \frac{1}{z^2} + \frac{1}{z} + \frac{1}{2i} + \frac{z}{3i} + \frac{z^2}{4i} + ...$

30. Use Cauchy's residue theorem to evaluate the integral $\int_C \frac{5z-2}{z(z-1)} dz$, where C is the circle $|z| = 2$, described counter clockwise.

(4 × 2 = 8)

Part D

Essay questions.

Answer any **two** *questions. Each question has weight* 4.

31. If $f(z)$ is analytic everywhere inside and on a simple closed contour C, taken in the positive sense, and z_0 is any point interior to C, then prove that $\int_C \frac{f(z)dz}{(z-z_0)^{n+1}} = \frac{2\pi i}{ni} f^n(z_0), n = 0, 1, 2, ...$

32. State prove Taylor's theorem.

33. Use residues to evaluate $\int_0^\infty \frac{\cos ax}{x^2+1} dx, a > 0.$

(2 × 4 = 8)