E 5861 (Pages : 3) Reg. No.....

Name.....

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, MARCH 2017

Sixth Semester

Core Course—COMPLEX ANALYSIS

(For B.Sc. Mathematics Model I and II)

[2013 Admission onwards]

Time : Three Hours Maximum : 80 Marks

Part A (Objective Type Questions)

Answer **all** *questions.*
Each question carries 1 *mark.*

1. Find the domain of definition of the function $f(z)=\frac{1}{z^2+1}$.
2. When is a function said to be analytic at a point z_0 ?
3. What is the real part of e^{iz} ?
4. Find the value of $\int_{|z|=1}\frac{dz}{z-4}$.
5. State Cauchy-Goursat theorem.
6. If C is the positive oriented unit circle $|z| = 1$, evaluate $\int_C \frac{e^z}{z^3}\,dz$.
7. Write the Maclaurin series expansion of e^z.
8. Find the Laurent series of $f(z)=\frac{1}{z-2}$ valid for $|z| > 2$.
9. Define residue of $f(z)$ at the isolated singular point z_0.
10. Find the residue at $z = 0$ of the function $f(z)=\frac{1}{z^2+z}$.

$(10 \times 1 = 10)$

Part B (Short Answer Questions)

Answer any **eight** *questions.*
Each question carries 2 *marks.*

11. Show that $f(z) = (z)^2$ is differentiable only at $z = 0$.
12. Prove that the function $f(z) = xy + iy$ is nowhere analytic.

Turn over

13. Find Log $(1 - i)$.

14. Define the cosine function of a complex variable z and show that it is an even function.

15. Evaluate $\int_C \frac{z+2}{z}\,dz$, where C is the circle $z - 2e^{i\theta} (\theta \le \theta \le 2\pi)$.

16. If C is any positive oriented simple closed contour surrounding origin, show that $\int_C \frac{dz}{z} = 2\pi i$.

17. Evaluate $\int_C \frac{dz}{z^2+1}$, where C is the positive oriented circle $|z| = 3$.

18. With the aid of remainders verify that $\sum_{n=0}^{\infty} z^n = \frac{1}{1-z}$ whenever $|z| < 1$.

19. State Laurent's theorem.

20. Find the nature of the singular point at $z_0 = 0$ of $f(z) = e^{1/z}$.

21. Define the improper integral $\int_{-\infty}^{\infty} f(x)\,dx$ and the Cauchy principal value of this integral.

22. State Jordan's lemma.

$(8 \times 2 = 16)$

Part C (Short Essay Questions)

Answer any **six** *questions.*
Each question carries 4 *marks.*

23. Prove that $f'(z) = 0$ everywhere in a domain D, then $f(z)$ is constant throughout D.

24. If $f(z) = u(x, y) + i\,v(x, y)$ is analytic in a domain D, prove that u and v are harmonic in D.

25. Prove that $\sin(z_1 + z_2) = \sin z_1 \cos z_2 + \cos z_1 \sin z_2$.

26. Let C be the arc of the circle $|z| = 2$ from $z = 2$ to $z = 2i$ that lies in the first quadrant. Without evaluating the integral, show that $\left|\int_C \frac{dz}{z^2-1}\right| \le \frac{\pi}{3}$.

27. State and prove the fundamental theorem of algebra.

28. Obtain the Taylor series $e^z = e\sum_{n=0}^{\infty} \frac{(3-1)^n}{n!} (|z-1| < \infty)$ for the function $f(z) = e^z$ by using (a) $f^{(n)}(1)$ $(n = 0, 1, 2, \ldots)$; (b) writing $e^z = e^{z-1}e$.

29. Expand $f(z) = \frac{-1}{(z-1)(z-2)}$ as a power series in the domains (a) $|z| < 1$; (b) $1 < |z| < 2$.

30. State and prove Cauchy's residue theorem.

31. Evaluate $\int_0^{\infty} \frac{dx}{(x^2+1)^2}$.

(6 × 4 = 24)

Part D (Essay Questions)

Answer any **two** *questions.*
Each question carries **15** *marks.*

32. (a) State and prove the chain rule for differentiating composite functions.

(b) Derive the Cauchy-Riemann equations.

33. (a) State and prove the Cauchy-Integral formula.

(b) If f is analytic everywhere inside and on a simple closed curve C, then for any point z inside of C, prove that

$$f'(z) = \frac{1}{2\pi i} \int_C \frac{f(s)\, dS}{(s-z)^2}.$$

34. (a) State and prove Taylor's theorem.

(b) Expand $f(z) = \frac{1}{z}$ into a Taylor series about the point $z_0 = 1$.

35. Use residues to evaluate :

(a) $\int_0^{\infty} \frac{\cos ax}{x^2+1}\, dx \quad (a > 0)$.

(b) $\int_0^{2\pi} \frac{d0}{1+a\sin 0} \quad (-1 < a < 1)$.

(2 × 15 = 30)