E 1685 (Pages : 3) Reg. No.................................

Name....................................

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, MARCH 2015

Sixth Semester

Core Course—REAL ANALYSIS

(For B.Sc. Mathematics Model I and II and B.Sc. Computer Applications)

Time : Three Hours Maximum Weight : 25

Part A

Answer **all** *questions.*
Each bunch of four questions has weight **1**.

I. 1 State Cauchy's general principle of convergence of a series.

2 What is a necessary condition for convergence of a series.

3 Show that the series $\sum \frac{1}{n}$ diverges.

4 What is a geometric series ?

II. 5 State Gauss's test.

6 Is every convergent series converges absolutely.

7 When we say that a function has a removable discontinuity at a point.

8 Show that if a function f is continuous at a point c, then $|f|$ is also continuous at c.

III. 9 Show that the function :

$$f(x) = \begin{cases} x \sin(1/x) & \text{when } x \neq 0 \\ 0 & \text{when } x = 0 \end{cases}$$

is continuous at $x = 0$.

10 State maximum-minimum theorem.

11 State Riemann's criterion for integrability.

12 What do you mean by a monotone function ?

IV. 13 If $|f|$ is integrable, then f is integrable, write True *or* False.

14 State Weierstrass's M test for uniform convergence.

15 Write whether the series $\sum \frac{\cos n\theta}{n^p}$ converges uniformly for $p > 1$.

Turn over

16 Show that the series $\sum \frac{x^n}{n^2}$ converges uniformly in $[-1,1]$.

$(4 \times 1 = 4)$

Part B

Answer any **five** *questions.*
Each question has weight 1.

17 Show that the series $1^2 + 2^2 + 3^2 + ...$ diverges to $+\infty$.

18 Examine the convergence of the series $\frac{1}{3} + \frac{1 \cdot 2}{3 \cdot 5} + \frac{1 \cdot 2 \cdot 3}{3 \cdot 5 \cdot 7} + ...$

19 Examine the convergence of the series $\frac{1}{1p} - \frac{1}{2p} + \frac{1}{3p} ....,\ p > 0$.

20 Show that the function

$$f(x) = \begin{cases} \frac{\sin 2x}{x}, & \text{when } x \neq 0 \\ 1, & \text{when } x = 0 \end{cases}$$

has a removable discontinuity at the origin.

21 Show that the function $f(x) = \frac{1}{x}$ is not uniformly continuous on $[0, 1]$.

22 Let $f : I \to R$ be a bounded function and P_1 and P_2 any two partitions on I, show that $L(p_1, f) \leq U(p_2, f)$.

23 Let a function f be defined on $[-1, 1]$ as

$$f(x) = \begin{cases} k, & \text{positive constant if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$$

show that f is integrable on $[-1, 1]$ and the value of the integral is $2k$.

24 Test for uniform convergence the series $\frac{2x}{1+x^2} + \frac{4x^3}{1+x^4} + \frac{8x^7}{1+x^8} + ..., -\frac{1}{2} \leq x \leq \frac{1}{2}$.

$(5 \times 1 = 5)$

Part C

Answer any **four** *questions.*
Each question has weight 2.

25 Show that the series

$$\frac{1}{(\log 2)^p} + \frac{1}{(\log 3)^p} + \ldots\ldots + \frac{1}{(\log n)^p} + \ldots\ldots \text{ diverges for } p > 0.$$

26 Show that the series $x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots\ldots$ converges absolutely for all values of x.

27 Show that the function $f(x)$ defined on R by

$$f(x) = \begin{cases} x & \text{when } x \text{ is irrational} \\ -x & \text{when } x \text{ is rational} \end{cases}$$

is continuous only at $x = 0$.

28 Show that the function f defined by

$$f(x) = \begin{cases} 0, & \text{when } x \text{ is rational} \\ 1, & \text{when } x \text{ is irrational} \end{cases}$$

is not integrable on any interval.

29 Compute $\int_{-1}^{1} f \, dx$, where $f(x) = |x|$.

30 State and prove Abel's test.

$(4 \times 2 = 8)$

Part D

Answer any **two** *questions.*
Each question has weight 4.

31 State and prove Leibnitz test.

32 Show that if a function f is continuous on a closed interval $[a, b]$, then it attains its bounds atleast once in $[a, b]$.

33 (i) Show that if f is integrable on $[a, b]$, then f^2 is also integrable on $[a, b]$.

(ii) If f_1 and f_2 are both integrable on $[a, b]$, then show that $f_1 f_2$ is also integrable on $[a, b]$.

$(2 \times 4 = 8)$