E 3403 (Pages : 3) Reg. No.................................

Name.................................

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, MARCH/APRIL 2012

Sixth Semester

Core Course—REAL ANALYSIS

(For B.Sc. Mathematics—Model-I,
B.Sc. Mathematics—Model-II and
B.Sc. Computer Applications)

Time : Three Hours Maximum Weight : 25

Part A (Objective Type Questions)

Answer **all** *the questions.*
Each bunch of 4 questions has weight **1**

I. 1 Show that the series $\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\cdots$ is not convergent.

2 State Cauchy's general principle of convergence for series.

3 Investigate the behaviour of the series whose n^{th} term is $\sin\left(\frac{1}{n}\right)$.

4 State Cauchy's root test.

II. 5 Define an alternating series.

6 When a series is said to be absolutely convergent ?

7 When a function f is said to be continuous at a point "C" of its domain ?

8 Show that $f(x)=\frac{\sin 2x}{x}, x \neq 0$

$= 1 \quad , \quad x = 0$

has removable discontinuity at the origin.

III. 9 State the intermediate value theorem.

10 When a function f defined on an internal I is said to be uniformly continuous on I ?

11 Define the norm of a partition P of $[a, b]$.

12 State Darboux's theorem.

IV. 13 State the fundamental theorem of Calculus.

14 Find the point-wise limit of the sequence of functions (f_n), where $f_n(x)=x^n, x \in [0,1]$.

Turn over

15 State Cauchy's criterion for uniform convergence of a sequence of functions (f_n) defined on $[a, b]$.

16 State Abel's test.

$(4 \times 1 = 4)$

Part B (Short Answer Questions)

Answer any **five** *questions.*
Each question has weight 1.

17 Show that the series $\frac{1}{(\log 2)^P} + \frac{1}{(\log 3)^P} + \ldots + \frac{1}{(\log n)^P} + \ldots$ diverges for $p > 0$.

18 Test for convergence the series $\Sigma \frac{n^2 - 1}{n^2 + 1} x^n, x > 0$.

19 Show that for any fixed value of x, the series $\sum_{n=1}^{\infty} \frac{\sin nx}{n^2}$ is convergent.

20 Show that the function f defined by

$$f(x) = x, \text{ if } x \text{ is rational}$$
$$= 0, \text{ if } x \text{ is irrational}$$

is continuous only at $x = 0$.

21 Show that $f(x) = x^2$ is uniformly continuous on $[-1, 1]$.

22 Show that a constant function k is integrable.

23 If P^* is a refinement of a partition P of $[a, b]$, then for a bounded function f, prove that $L(P^*, f) \geq L(P, f)$.

24 Show that the sequence $\{f_n\}$, where $f_n(x) = \frac{nx}{1 + n^2 x^2}$ is not uniformly convergent on any interval containing zero.

$(5 \times 1 = 5)$

Part C (Short Essay Questions)

Answer any **four** *questions.*

Each question has weight 2.

25 If ΣU_n is a positive term series, such that $\lim_{n \to \infty} n\left(\frac{U_n}{U_{n+1}} - 1\right) = l$, then prove that the series converges if $l > 1$.

26 Show that $\lim_{n \to \infty} \frac{m(m-1)\ldots(m-n+1)}{(n-1)!} x^n = 0$, where $|x| < 1$ and "m" is any real number.

27 If a function f is continuous on $[a, b]$, then prove that it attains its bounds atleast once in $[a, b]$.

28 If f_1 and f_2 are two bounded and integrable functions on $[a, b]$, then prove that $f_1 - f_2$ is also integrable on $[a, b]$.

29 If a function f is monotonic on $[a, b]$, then prove that it is integrable on $[a, b]$.

30 State and prove Weierstrass's M-test.

$(4 \times 2 = 8)$

Part D (Essay Type)

Answer any **two** *questions.*
Each question has weight 4.

31 Prove that a positive term series $\Sigma \frac{1}{n^p}$ is convergent if and only if $p > 1$.

32 If a function f is continuous on $[a, b]$ and $f(a)$ and $f(b)$ are of opposite signs, then prove that there exists atleast one point $\alpha \in]a, b[$ such that $f(\alpha) = 0$.

32 If f is bounded and integrable on $[a, b]$, then :

(a) Prove that $|f|$ is also bounded and integrable on $[a, b]$.

(b) Prove that f_2 is also integrable on $[a, b]$.

$(2 \times 4 = 8)$