E 2172 (Pages : 4) Reg. No.....

Name.....

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, MAY 2015

Second Semester

Complementary Course—OPERATIONS RESEARCH – DUALITY TRANSPORTATION AND ASSIGNMENT PROBLEMS

(For B.Sc. Mathematics Model II)

[2013 Admission onwards]

Time : Three Hours Maximum : 80 Marks

Part A (Short answer questions)

Answer *all* questions from this part. Each question carries 1 mark :

1. What is the relationship between the co-efficient matrix of the primal problem and that of the dual problem ?
2. If there are m constraints and n variables in a primal problem, then how many constraints and variables are there in the dual problem.
3. Write the dual of the problem :

$$\begin{aligned} &\text{Maximize } Z = x_1 - x_2 + 3x_3 \\ &\text{subject to} \quad x_1 + x_2 + x_3 \le 10 \\ &\qquad 2x_1 - 0x_2 - x_3 \le 2 \\ &\qquad 2x_1 - 2x_2 - 3x_3 \le 6, \quad x_1, x_2, x_3 \ge 0. \end{aligned}$$

4. What is a triangular basis ?
5. What is a balanced transportation problem ?
6. What is meant by degenerate solution of a transportation problem ?
7. Define loop of a transportation table.
8. What are occupied cells ?
9. Define the term optimal solution with reference to a transportation problem.
10. Define the term feasible solution with reference to a transportation problem.

$(10 \times 1 = 10)$

Turn over

Part B (Brief answer questions)

Answer any *eight* questions. Each question carries 2 marks :

11. Write the dual problem of the following primal problem :—

Minimize $Z = x_1 - 3x_2 - 2x_3$

Subject to constraints

$$3x_1 - x_2 + 2x_3 \leq 7$$
$$2x_1 - 4x_2 \geq 12$$
$$-4x_1 + 3x_2 + 8x_3 = 10$$ and x_3 unrestricted.

12. How can the optimal solution of the primal be obtained from the optimal solution of the dual.
13. When is it advantageous to solve an L. P. P. by dual simplex method ?
14. State a transportation problem.
15. Write the general transportation problem in which the total demand exceeds the total supply.
16. What is degeneracy in a transportation problem ?
17. Write the matrix associated with a transportation problem involving two origins and three destinations.
18. When does a transportation problem has a unique solution ?
19. State the assignment problem.
20. What is an unbalanced assignment problem ?
21. Write an example of an assignment problem.
22. State travelling salesman problem.

$(8 \times 2 = 16)$

Part C (Short essay type questions)

Answer any *six* questions. Each question carries 4 marks :

23. Show that dual of the dual is the primal.
24. For the problem minimize $x_1 + x_2$

Subject to $2x_1 + x_2 \geq 8$

$$3x_1 + 7x_2 \geq 21$$
$$x_1 \geq 0, x_2 \geq 0.$$

(i) Find the dual ; (ii) Solve the primal and dual graphically.

25. What are the advantages of duality ?

26. Find the initial basic feasible solution of the transportation problem whose cost matrix is given below :

		Destination				
		A	B	C	D	*Available*
	I	1	5	3	3	34
	II	3	3	1	2	15
Origin	III	0	2	2	3	12
	IV	2	7	2	4	19
Required		21	25	17	17	

27. Solve the following transportation problem for minimum cost starting with the degenerate solution $x_{12} = 30, x_{21} = 40, x_{32} = 20, x_{43} = 60$.

	D_1	D_2	D_3	
O_1	4	5	2	30
O_2	4	1	3	40
O_3	3	6	2	20
O_4	2	3	7	60
	40	50	60	

28. Describe the computational procedure of optimality test in a transportation problem.

29. Explain the trans-shipment problem.

30. Explain the difference between a transportation problem and an assignment problem.

31. The following table presents the cost of assigning three jobs to three workers. Workout the optimal assignment :-

		Job		
		A	B	C
	1	4	7	8
Workers	2	5	3	2
	3	6	5	4

(6 × 4 = 24)

Part D

Answer any *two* questions. Each question carries 15 marks :

32. Use dual simplex method to solve the L.P.P,

Maximize $Z = 2x_1 + x_2 + x_3$

Subject to the constraints

$$4x_1 + 6x_2 + 3x_3 \leq 8$$
$$x_1 - 9x_2 + x_3 \leq -3$$
$$-2x_1 - 3x_2 + 5x_3 \leq -4$$
$$x_1, x_2, x_3 \geq 0.$$

33. A caterer needs clean table covers every day for six days to meet a contract according to the following schedule :

Days	:	1	2	3	4	5	6
Number of covers	:	50	60	80	70	90	100

The cost of a new cover is Rs. 20 while washing charges are Rs. 1 for return on the fourth day or later, Rs. 2 for return on the third day and Rs. 3 for the next day. Find the minimum cost schedule for the purchase and washing of table covers, assuming that after the end of the contract the covers are rejected.

34. A sales man has to visit five cities A, B, C, D and E. The distances (in hundred kilometres) between the five cities are as follows :—

		To city				
		A	B	C	D	E
From city	A	_	7	6	8	4
	B	7	_	8	5	6
	C	6	8	_	9	7
	D	8	5	9	_	8
	E	4	6	7	8	_

35. Prove that the value of the objective function $f(x)$ for any feasible solution of the primal is not less than the value of the objective function $\phi(y)$ for any feasible solution of the dual. Also prove that $\min f(x) \geq \max \phi(y)$.

(2 × 15 = 30)