E 3143 (Pages : 4) Reg. No..............................

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B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, APRIL 2012

Second Semester

Complementary Course 2—DUALITY, TRANSPORTATION AND ASSIGNMENT PROBLEMS

(For B.Sc. Mathematics Model II Programme)

Time : Three Hours Maximum Weight : 25

Part A (Objective Type Questions)

Answer all *questions.*
Each bunch of 4 questions has weight 1.

I. 1 How many terms are there in the objectives function of the dual of a linear programming problem with n variables and m constraints ?

2 If in a primal problem, the j^{th} variable x_j is unrestricted, then what can we say about the j^{th} constraint of its dual ?

3 How many feasible solutions are there for the dual of a linear programming problem if the primal problem is feasible and it has an unbounded optimum ?

4 If x_j is positive in the optimal solution of the primal, then what is the value of the corresponding dual slack variable y_{m+j} in the optimal solution of its dual ?

II. 5 What is meant by a balanced transportation problem ?

6 What is the condition for which a basis to be triangular ?

7 Suppose that in a transportation problem, surplus left at the sources after all the demands are met. Then what is the relation between $\sum a_i$ and $\sum b_j$ with the standard notations ?

8 What will we do in the solution of a transportation problem if the actual supply is short of the demand ?

III. 9 Consider a balanced transportation problem with $m = 4$, $n = 6$. What is the rank of the transportation matrix ?

10 Give an example of a situation in which a loop occur in a transportation array.

11 How many non-zero entries are there in the second column of the transportation matrix of a transportation problem with 3 sources and 4 sinks ?

12 Write the general form of an objective function of a transportation-cum-transshipment problem.

IV. 13 Which of the following statements is true ?

Statement A : Every assignment problem is a transportation problem.

Statement B : Every transportation problem is an assignment problem.

14 What is the number of basic variables in a basic feasible solution of an assignment problem with n workers and n different jobs ?

Turn over

15 Write the general form of the constraints for an assignment problem.

16 What is the relation between the outflow and inflow at a source for a transportation problem with transshipment ?

(4 × 1 = 4)

Part B (Short Answer Type Questions)

Answer any **five** *questions.*
Each question has weight 1.

17 Give an example of a linear programming problem in both its primal and dual forms.

18 Write *two* applications of duality.

19 What is the mathematical model of the general transportation problem ?

20 Write a short note on transportation-cum-transhipment problem.

21 State the caterer problem.

22 What is the difference between a transportation array and a transportation matrix ?

23 State the generalized transportation problem.

24 Write the first step of the transportation algorithm.

(5 × 1 = 5)

Part C (Short Essay Type Questions)

Answer any **four** *questions.*
Each question has weight 2.

25 Solve the following problem by solving its dual graphically :

Maximize : $y_1 + y_2 + y_3$

subject to : $2y_1 + y_2 + 2y_3 \le 2$

$4y_1 + 2y_2 + y_3 \le 2$

$y_j \ge 0$ for $j = 1, 2, 3$.

26 Explain with examples the applications of linear programming.

27 Solve the transportation problem for minimum cost with the cost coefficients, demands and supplies as given in the following table :

	D_1	D_2	D_3	D_4	
O_1	1	2	−2	3	70
O_2	2	4	0	1	38
O_3	1	2	−2	5	32
	40	28	30	42	

28 A farmer has three farms A, B, C which need respectively 100, 300 and 50 units of water annually. The canal can supply 150 units and tubewell 200 units while the balance is left at the mercy of Rain God. The following table shows the cost per unit of water in a dry year when the rains totally fail, the third row giving the cost of failure of rain. Find how the canal and tubewell water should be utilized to minimize the total cost.

	A	B	C	
Canal	3	5	7	150
Tubewell	6	4	10	200
Failure rain	8	10	3	100
	100	300	50	

29 Food bags have to be lifted by three different types of aircraft A_1, A_2, A_3 from an airport and dropped in flood affected villages V_1, V_2, V_3, V_4, V_5. The quantity of food that can be carried in one trip by aircraft A_i to village V_j is given in the following table. The total number of trips that A_i can make in a day is given in the last column. The number of trips possible each day to village V_j is given in the last row. Find the number of trips each aircraft should make on each village so that the total quantity of food transported in a day is maximum.

	V_1	V_2	V_3	V_4	V_5	
A_1	10	8	6	9	12	50
A_2	5	3	8	4	10	90
A_3	7	9	6	10	4	60
	100	80	70	40	20	

30 Solve the following problem of transportation with transhipment with sources S_1, S_2 sinks D_1, D_2 and junction J for minimum cost.

Minimum cost

		S_1	S_2	J	D_1	D_2
Transportation cost		4	3	1	3	5
capacity		60	40	–	35	45
	S_1	–	4	3	10	5
Transportation	S_2	4	–	2	5	6
cost	J	4	2	–	8	7
	D_1	11	4	6	–	4
	D_2	5	7	5	4	–

$(4 \times 2 = 8)$

Part D (Essay Type Questions)

Answer any **two** *questions.*
Each question has weight 4.

31 Solve the following problem by the Dual simplex method :

Minimize : $2x_1 + 3x_2$

subject to : $2x_1 + 3x_2 \leq 30$,

$x_1 + 2x_2 \geq 10$

$x_1 \geq 0, x_2 \geq 0$.

32 Solve the following transportation problem for minimum cost starting with the degenerate solution :

$x_{12} = 30, x_{21} = 40, x_{32} = 20, x_{43} = 60$

	D_1	D_2	D_3	
O_1	4	5	2	30
O_2	4	1	3	40
O_3	3	6	2	20
O_4	2	3	7	60
	40	50	60	

33 A batch of four jobs can be assigned to five different machines. The set-up time for each job on each machine is given in the following table. Find an optimal assignment of jobs to machines which will minimize the total set-up time.

Machines

		1	2	3	4	5
Jobs	1	10	11	4	2	8
	2	7	11	10	14	12
	3	5	6	9	12	14
	4	13	15	11	10	7

(2 × 4 = 8)