

E 3097

(Pages : 3)

Reg. No.....

Name.....

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, APRIL 2012

Second Semester

Core Course 2—ANALYTIC GEOMETRY, TRIGONOMETRY AND MATRICES

(For B.Sc. Mathematics Model I, B.Sc. Mathematics Model II and B.Sc. Computer Applications)

Time : Three Hours

Maximum Weight : 25

Part A (Objective Type Questions)

Answer all questions.

A bunch of 4 questions has weight 1.

- I. 1 What is the parametric representation of a point on the parabola $y^2 = -16x$?
2 What is the x coordinate of the point of intersection of the tangents at $t_1 = 8$ and $t_2 = 0$ of the parabola $y^2 = -x$?
3 Write the equation of the directrix of the parabola $y^2 = 3x$.
4 If S, S' are the foci of the ellipse $\frac{x^2}{36} + \frac{y^2}{9} = 1$ and P is a point on the ellipse such that $SP = 3$, what is $S'P$?
- II. 5 What is $m_1 m_2$ if $y = m_1 x$ and $y = m_2 x$ are conjugate diameters of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$?
6 What is the eccentricity of the hyperbola $xy = 1$?
7 What is the angle between the asymptotes a rectangular hyperbola ?
8 Which curve is represented by the polar equation $\frac{10}{r} = 3 \cos \theta + 4 \sin \theta + 5$?
- III. 9 For what values of θ is r infinite in value for the conic $r = \frac{10}{1 - 2 \cos \theta}$?
10 What is the period of the hyperbolic sine function ?
11 Find a root of the equation $x^6 - 1 = 0$, other than 1 ?
12 Write a polar representation of -1 .

- IV. 13 What is the rank of matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & \frac{1}{2} & 0 \end{bmatrix}$?

Turn over

- 14 Give an elementary matrix obtained from $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.
- 15 What is the relation between rank of A and number of unknowns n if the system of homogeneous equations $AX = 0$ has an infinite number of solutions?
- 16 Give an example of a 2×2 matrix A such that sum of its eigenvalues is 1.

(4 × 1 = 4)

Part B (Short Answer Type Questions)*Answer any five questions.**Each question has weight 1.*

- 17 Prove that in a parabola if the normal at P meets the axis in G, then $SG = GP$, where S is the focus of the parabola.
- 18 Prove that the sum of the squares of two conjugate semi diameters of an ellipse is constant.
- 19 Find the equation of the normal to the rectangular hyperbola $xy = c^2$ at the point 't' using the equation of the normal at 't'.
- 20 What is the minimum value of r for the conic $\frac{l}{r} = 1 + \cos \theta$?
- 21 Separate $\sinh(2 + 3i)$ into real and imaging parts.
- 22 Prove that $\sin(iy) = i \sinh(y)$.

- 23 Write the normal form of the matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$.

- 24 State Cayley-Hamilton theorem.

(5 × 1 = 5)

Part C (Short Essay Type Questions)*Answer any four questions.**Each question has weight 2.*

- 25 Prove that the eccentric angles of the ends of a pair of conjugate diameters of an ellipse differ by a right angle.
- 26 Show that the normal to the rectangular hyperbola $xy = c^2$ at the point $P\left(ct, \frac{c}{t}\right)$ meets the curve again at the point $Q\left(\frac{-c}{t^3}, -ct^3\right)$.

27 Find the locus of the foot of the perpendicular drawn from the pole to the tangents to the circle $r = 2a \cos \theta$.

28 Sum to n terms, and to infinity the series $1 + c \cos \alpha + c^2 \cos 2\alpha + \dots$

29 Find all non-trivial solutions of $x_1 + 2x_2 + 3x_3 = 0$
 $2x_1 + x_2 + 3x_3 = 0$
 $3x_1 + 2x_2 + x_3 = 0.$

30 Using Cayley-Hamilton theorem, find A^3 for the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 5 \\ 1 & 2 & 0 \end{bmatrix}$.

(4 × 2 = 8)

Part D (Essay Type Questions)

Answer any two questions.

Each question has weight 4.

31 The polar of a point P with respect to the parabola $y^2 = 4ax$ meets the curve in Q and R. Show that if P lies on the line $lx + my + n = 0$ then the middle point of QR lies on the parabola $l(y^2 - 4ax) + 2a(lx + my + n) = 0$.

32 Factorize $x^6 - 2x^4 \cos 60^\circ + 1$.

33 Reduce the matrix $A = \begin{bmatrix} 0 & 2 & 3 & 4 \\ 2 & 3 & 5 & 4 \\ 4 & 8 & 13 & 12 \end{bmatrix}$ to normal form.

(2 × 4 = 8)