E 2137 (Pages : 3) Reg. No.................................

Name.................................

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, APRIL 2016

Fourth Semester

FOURIER SERIES, DIFFERENTIAL EQUATIONS NUMERICAL ANALYSIS AND ABSTRACT ALGEBRA

Complementary Course to Physics/Chemistry/Petrochemicals /Geology, Food Science and Quality Control and Computer maintenance and Electronics)

[2013 Admission onwards]

Time : Three Hours Maximum : 80 Marks

Part A

Answer **all** *questions.*
Each question carries 1 mark.

1. Find the fundamental period of $\cos\left(\frac{2\pi x}{k}\right)$.
2. Write the Legendre polynomial of degree n.
3. Eliminate the constants a and b from $z = (x + a)(y + b)$.
4. Write the Lagrange's partial differential equation.
5. Distinguish between an ordinary differential equation and a partial differential equation.
6. Find the relative error of the number 8.6 if both of its digits are correct.
7. Round off 36.46235 to four significant figures.
8. What are the generators of z_4?
9. Every group G is isomorphic to a subgroup of S_G. Write true *or* false.
10. What are the units of z_{14}?

$(10 \times 1 = 10)$

Part B

Answer any **eight** *questions.*
Each question carries 2 marks.

11. Sketch the function :

$$f(x) = \begin{cases} x & \text{if } -\pi \le x \le 0 \\ 0 & \text{if } 0 \le x \le \pi. \end{cases}$$

12. Write the Bessel functions $J_0(x)$ and $J_1(x)$.
13. Eliminate the arbitrary function f from the equation $z = x + y + f(xy)$.

Turn over

14. Find $\frac{\partial(F \cdot G)}{(y, z)}$ where $F = ax^2 + by^2 + cz^2 - 1$, $G = x + y + z - 1$.

15. Eliminate the arbitrary function f from the equation $z = f(x - y)$.

16. Given $u = 5xy^2/z^3$, find $(E_R)_{max}$.

17. Obtain a root, correct to three decimal places for $x^3 - x^2 - 1 = 0$ by bisection method.

18. Define the term absolute error. If $C = 15300 \pm 100$, find the maximum value of the absolute error in C^3.

19. Find an approximate root of $\cos x = 3x - 1$ using the iterative method.

20. Prove that the identity element and the inverse of each element are unique in a group.

21. Determine whether $\{(1, 1, 0), (1, 0, 1), (0, 1, 1)\}$ is a basis for R^3 over R.

22. Given $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 4 & 5 & 6 & 2 \end{pmatrix}$ and $\mu = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 2 & 4 & 3 & 1 & 6 \end{pmatrix}$. Find $\mu\sigma^2$.

$(8 \times 2 = 16)$

Part C

Answer any **six** *questions.*
Each question carries 4 marks.

23. Find the Fourier series for the function $f(x) = x^2, -\pi < x < \pi$.

24. Show that $P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n}\left[(x^2 - 1)^n\right]$.

25. Find the integral curves of $\frac{dx}{y(x+y) + az} = \frac{dy}{x(x+y) - az} = \frac{dz}{z(x+y)}$.

26. Find the general solution of the differential equation :

$$x^2 \frac{\partial z}{\partial x} + y^2 \frac{\partial z}{\partial y} = (x + y) z.$$

27. Find the Fourier cosine series as well as Fourier sine series of $f(x) = e^x$, $0 < x < L$.

28. Use the method of false position to a root, correct to three decimal places of $x^3 - x - 4 = 0$.

29. Use the method of iteration to find a positive root between 0 and 1 of the equation $xe^x = 1$.

30. Let G be a group and let $a \in G$. Show that $H = \{a^n, n \in Z\}$ is a subgroup of G and is the smallest subgroup of G that contains a.

31. Show that the set R of all real numbers is a vector space over itself under usual addition and scalar multiplication.

$(6 \times 4 = 24)$

Part D

Answer any **two** *questions.*
Each question carries 15 marks.

32. (a) Find the Fourier series of $f(x) = \frac{x^2}{2}, -\pi < x < \pi$.

(b) Show that $1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \ldots = \frac{\pi^2}{6}$.

(c) Show that $J_0'(x) = -J_1(x)$.

33. (a) Find the partial differential equation of all spheres whose centres lie on the z-axis.

(b) Find the integral curves of the equations $\frac{dx}{x+z} = \frac{dy}{y} = \frac{dz}{z+y^2}$.

(c) Find the general integral of $z(xp - yq) = y^2 - x^2$.

34. (a) Use Newton-Raphson method to obtain a root correct to three decimal places $x^3 - 5x + 3 = 0$.

(b) Apply quotient difference method to obtain an approximate root of $x^3 - x^2 - 2x + 1 = 0$.

35. (a) Define the permutation group S_3. Find all subgroups of S_3 and draw the lattice diagram.

(b) If V is a vector space over F, show that $0\alpha = 0, a0 = 0$ and $(-a)\alpha = a(-\alpha) = -(a\alpha)$ for all $a \in F$ and $\alpha \in V$.

$(2 \times 15 = 30)$