E 2852 (Pages : 4) Reg. No...............................

Name...................................

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, MARCH/APRIL 2012

Fourth Semester

Complementary Course 4—NON-LINEAR PROGRAMMING

[For Model–II B.Sc. Mathematics)

Time : Three Hours Maximum Weight : 25

Part A (Objective Types Questions)

Answer **all** *questions.*
Each bunch of 4 questions has weight 1.

1. When we call, a vector $X \in E_n$, a mixed integer vector ?
2. Name one method that we use to find the solution of an integer linear programming problem.
3. Write one disadvantage of the cutting plane method.
4. Give one situation in which branching terminates in the branch and bound method.
5. Write one situation where we use the 0 – 1 variables.
6. Give one strategy for determining a lower bound while using branch and bound method.
7. When we say that a subproblem is pruned while using branch and bound method for solving an integer linear programming problem ?
8. What is the relation between the set of all feasible solutions of an integer linear programming problem and its associated linear programming problem ?
9. If the objective function $f(X)$ is convex, then what can we say about $-f(X)$?
10. When a mathematical programming problem is said to be a convex programming problem ?
11. What is the condition for which (X_0, Y_0) to be a saddle point of $F(X, Y)$?
12. Give an example of a non-linear programming problem.
13. If (X_0, Y_0) is a saddle point of $F(X, Y)$ and $F(X_0, Y_0) = 21$, find $\min_X \max_Y F(X, Y)$.
14. Give an example of a positive definite quadratic form.
15. Give one condition for which the objective function

    $f(X) = PX + X'CX$ of a quadratic programming problem cannot have an unbounded optimum.
16. Give an example of a separable function. $(4 \times 1 = 4)$

Turn over

Part B (Short Answer Type Questions)

Answer any **five** *questions.*
Each question has weight 1.

17. What are the two strategies involved in the branch and bound method ?
18. When we say that a problem is fathomed with respect to the branch and bound method ?
19. What are the constraints of the subproblems of an integer linear programming problem with respect to the branch and bound method ?
20. Define the Lagrangian function.
21. What is the relation between a saddle point of $F(X, Y)$ and a minimal point of $F(X)$ with respect to a convex programming problem ?
22. Using an example exhibit the formation of dual from its primal problem in mathematical programming.
23. Write the Kuhn-Tucker conditions.
24. Why we prefer to find a lower bound than to find the minimum of the objective function of an integer linear programming problem while using branch and bound method ?

(5 × 1 = 5)

Part C (Short Essay Type Questions)

Answer any **four** *questions.*
Each question has weight 2.

25. Solve the following problem by cutting plane method.

Minimise $3x_1 - x_2$

subject to $-10x_1 + 6x_2 \leq 15,$

$14x_1 + 18x_2 \geq 63,$

x_1, x_2 non-negative integers.

26. Solve the following problem using branch and bound method.

Minimise $f = 3x_4 + 4x_5 + 5x_6$

subject to $2x_1 + 2x_4 - 4x_5 + 2x_6 = 3$

$2x_2 + 4x_4 + 2x_5 - 2x_6 = 5$

$x_3 - x_4 + x_5 + x_6 = 4$

$x_1, x_2, ..., x_6 \geq 0$; x_1, x_2 integers.

27. Describe the branch and bound method for solving an integer linear programming problem.

28. Give example for a linear programming problem, a nonlinear programming problem which is not of convex programming and a convex programming problem and describe one method each to solve them.

29. Prove that if $F(X, Y)$ has a saddle point (X_0, Y_0) for every $Y \geq 0$ then X_0 is a minimal point of $f(X)$ subject to the constraints $G(X) \leq 0$.

30. Solve the following by Kuhn-Tucker conditions :

$$\text{Maximize} \quad x_1$$

$$\text{subject to} \quad (x_1 - 4)^2 + x_2^2 \leq 16$$

$$(x_1 - 3)^2 + (x_2 - 2)^2 = 13$$

$(4 \times 2 = 8)$

Part D (Essay Type Questions)

Answer any **two** *questions.*
Each question has weight 4.

31. Solve the following problem.

$$\text{Maximize} \quad x_1 + 2x_2$$

$$\text{subject to} \quad 5x_1 + 7x_2 \leq 21,$$

$$-x_1 + 3x_2 \leq 8;$$

x_1, x_2 non-negative integers.

32. Solve the following problem graphically :

$$\text{Minimize} \quad (x_1 - 4)^2 + (x_2 - 4)^2$$

subject to the contraints $x_1 + x_2 \leq 6,\ x_1 - x_2 \leq 1,\ 2x_1 + x_2 \geq 6,\ \frac{1}{2}x_1 - x_2 \geq -4,\ x_1 \geq 0, x_2 \geq 0$

33. Solve by the method of quadratic programming.

Minimize $f(X) = -x_1 - x_2 - x_3 + \frac{1}{2}\left(x_1^2 + x_2^2 + x_3^2\right)$

subject to $g_1(X) = x_1 + x_2 + x_3 - 1 \leq 0.$

$g_2(X) = 4x_1 + 2x_2 - \frac{7}{3} \leq 0.$

$x_1, x_2, x_3 \geq 0.$

(2 × 4 = 8)