**E 6187** (Pages : 4) Reg. No.................................

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# B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, MARCH 2017

## Fourth Semester

Complementary Course—OPERATIONS RESEARCH-NON-LINEAR PROGRAMMING

[For B.Sc. Mathematics Model II]

(2013 Admission onwards)

Time : Three Hours Maximum Marks : 80

### Part A

*Answer* **all** *questions.*
*Each question carries* 1 *mark.*

1. What is integer programming ?
2. Define the term pruned in branch and bound method.
3. When is a branch and bound method fathomed ?
4. Who was the first person proposed the method of cutting plane to solve an I.L.P ?
5. What is meant by an optimal solution in an integer linear programming problem ?
6. Define a convex set.
7. Show that $f(x) = x^2$ is a convex function.
8. Define saddle point.
9. Find $\nabla f(X)$ for the function $f(X) = x_1^2 + 3x_1x_2 - 4x_1^2 + 4x_1 + 5x_1x_2 - x_3^2$.
10. Does the intersection of two convex sets convex ? Justify.

(10 × 1 = 10)

### Part B

*Answer any* **eight** *questions.*
*Each question carries* 2 *marks.*

11. What are the disadvantages of cutting plane method ?
12. Maximize $x_1 + x_2$ subject to $7x_1 - 6x_2 \le 5, 6x_1 + 3x_2 \ge 7, -3x_1 + 8x_2 \le 6$ ; $x_1, x_2$ non-negative integers.

Turn over

13. Minimize $f = x_1^2 + x_2^2$ subject to $g = (x_1 - 1)^3 - x_2^2 \geq 0$.

14. What is quadratic programming ?

15. When is a function seperable ? Check whether the function

$$f(x_1, x_2, x_3) = x_1^3 - 2x_1^2 + 4x_1 + 3x_2^4 - 4x_2 + 5\sin(x_3 + 2)$$ is seperable.

16. Write down the Kuhn-Tucker conditions.

17. Write the Lagrangian for the problem, minimize $f(X) = -x_1 - x_2 - x_3 + \frac{1}{2}\left(x_1^2 + x_2^2 + x_3^2\right)$, subject to $g_1(X) = x_1 + x_2 + x_3 - 1 \leq 0$, $g_2(X) = 4x_1 + 2x_2 - \frac{7}{3} \leq 0$, $x_1, x_2, x_3 \geq 0$.

18. Solve graphically maximize $(x_1 - 4)^2 + (x_2 - 4)^2$, subject to the constraints

$$x_1 + x_2 \leq 6,\ x_1 - x_2 \leq 1,\ 2x_1 + x_2 \geq 6,\ \frac{1}{2}x_1 - x_2 \geq -4,\ x_1 \geq 0,\ x_2 \geq 0.$$

19. Formulate the knapsack problem as an ILP. There are $n$ objects, $j = 1, 2, \ldots, n$, whose weights are $w_j$ and values $v_j$. They have to be chosen to be packed in a knapsack so that the total value of the of the objects chosen is maximum subject to their total weight not exceeding W.

20. Describe 0–1 problem.

21. What is the relation between saddle point of $F(X, Y)$ and minimal point of $f(X)$.

22. Write down the general form of the convex programming problem.

(8 × 2 = 16)

## Part C

*Answer any* **six** *questions.*
*Each question carries 4 marks.*

23. Explain branch and bound method.

24. Use cutting plane method, Maximize $3x_1 - x_2$ subject to $-10x_1 + 6x_2 \leq 15$, $14x_1 + 18x_2 \geq 63$ ; $x_1, x_2$ integers.

25. Use branch and bound method, minimize $9x_1 + 10x_2$, subject to

$0 \le x_1 \le 10, 0 \le x_2 \le 8, 3x_1 + 5x_2 \ge 45$ ; $x_2$ integer.

26. Maximize $2x_1 + 5x_2$ subject to $0 \le x_1 \le 8, 0 \le x_2 \le 8$, and either $4 - x_1 \ge 0$ or $4 - x_2 \ge 0$.

27. Suppose that $X_0$ be a solution of the convex programming problem, and let X be the set of points with $G(X) \le 0$, be not empty. Prove that there exist a vector $Y_0 > 0$ in $E_m$ such that $f(X) + Y_0' G(X) \ge f(X_0)$.

28. Find the minimum of $f(X) = (x_1 + 1)^2 + (x_2 - 2)^2$ subject to

$g_1(X) = x_1 - 2 \le 0, g_2(X) = x_2 - 1 \le 0, x_1 \ge 0, x_2 \ge 0.$

29. Maximize $f(x_1, x_2) = 2x_1 + 3_2^4 + 4$, subject to $g(x_1, x_2) = 4x_1 + 2x_2^2 \le 16, x_1 \ge 0, x_2 \ge 0.$

30. Use Kuhn-tucker conditions to find the extrema $(x_1 - 4)^2 + (x_2 - 3)^2$ of subject to $36(x_1 - 2)^2 + (x_2 - 3)^2 \le 9.$

31. Verify the validity of the primal and dual relationship in linear programming on the basis of Kuhn-Tucker conditions.

$(6 \times 4 = 24)$

## Part D

*Answer any* **two** *questions.*
*Each question carries* 15 *marks.*

32. Use branch and bound method :

Maximize $13x_1 + 3x_2 + 3x_3$
subject to $7x_1 + 6x_2 - 3x_3 \le 8,$
$7x_1 - 3x_2 + 6x_3 \le 8;$
$x_1, x_2, x_3$

subject to non-negative integers

33. Solve the knapsadk problem with the following data, knapsack capacity $W = 12$.

| Object | Weight | Value |
|---|---|---|
| $j$ | $w_j$ | $v_j$ |
| 1 | 2 | 10 |
| 2 | 2 | 14 |
| 3 | 3 | 18 |
| 4 | 6 | 48 |
| 5 | 8 | 80 |

34. Maximize $f - 4(x_1 - 6)^2 + (x_2 - 2)^2$,

subject to $3(x_1 + 1)^2 + 6x_2 \leq 12, x_1 \geq 0, x_2 \geq 0$.

35. Solve by the method of quadratic programming

Minimize $-6x_1 + 2x_1^2 - 2x_1x_2 + 2x_2^2$

subject to $x_1 + x_2 \leq 2, x_1 > 0, x_2 \geq 0$.

$(2 \times 15 = 30)$