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B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, MARCH 2015

Fourth Semester

Core Course—VECTOR CALCULUS, THEORY OF EQUATIONS AND NUMERICAL METHODS

(Common for Mathematics Model I, II and B.Sc. Computer Applications)

[2013 Admissions]

Time : Three Hours Maximum : 80 Marks

Part A

Answer **all** *the questions.*
Each question carries 1 mark.

1. Find the parametric equation of a line through the origin and parallel to the vector $2j + k$.
2. Find the unit vector tangent to the curve $r(t) = (6 \sin 2t)\, i + (6 \cos 2t)\, j + 5t\, k, 0 \le t \le \pi$.
3. Find the curvature of $r(t) = ti + (\ln \cos t)\, j, -\pi/2 < t < \pi/2$.
4. State Gauss's divergence theorem.
5. Define the gradient field of a differentiable function $f(x, y, z)$.
6. Define a reciprocal equation.
7. State Descarte's rule of signs.
8. If α, β, γ are the roots of $x^3 - px^2 + qx - r = 0$. Find the value of $\sum \alpha^2$.
9. Give an example of a transcendental function.
10. What are algebraic functions.

$(10 \times 1 = 10)$

Part B

Answer any **eight** *questions.*
Each question carries 2 marks.

11. Find the derivative of $f(x, y, z) = x^3 - xy^2 - z$. at (1, 1, 0) in the direction of $2i - 3j + 6k$.
12. Write the equation of hyperboloid of one-sheet and describe the sections cut out by the co-ordinate planes.

Turn over

13. Find the torsion τ for the helix $r(t) = (a \cos t)i + (a \sin t)j + bt\,k,\ a\,;\,b \geq 0,\ a^2 + b^2 \neq 0$.

14. Evaluate $\int_C f(x, y, z)\,ds,$, where $p(x, y, z) = x - 3y^2 + z$, C is the line segment joining the origin and the point (1, 1, 1).

15. Find the work done by the force $F = zi + xj + yk$ over the curve $r(t) = (\sin t)i + (\cos t)j + tk$, $0 \leq t \leq \pi$, in the direction of increasing t.

16. Show that $F = (2x - 3)i - zj + (\cos z)k$ is not conservative.

17. Evaluate the integral $\oint_C xydy - y^2\,dx$ where C is the square cut from the first quadrant by the lines $x = 1$ and $y = 1$.

18. Solve the equation $27x^3 + 42x^2 - 28x - 8 = 0$. Whose roots are in geometric progression.

19. Find the equation whose roots are the roots of $2x^5 - 9x^3 + 4x + 3 = 0$ each increased by 2.

20. If $\alpha, \beta, \gamma, \delta$ are the roots of $x^4 + px^3 + qx^2 + rx + s = 0$ evaluate $\Sigma \alpha^2 \beta\gamma$.

21. Given that the equation $x^{2.2} = 69$ has a roof between 5 and 8. Use the method of regula-falsi to determine if.

22. Set up Newton-Raphson iteration formula for computing the square roof of a given positive number.

$(8 \times 2 = 16)$

Part C

Answer any **six** *questions.*
Each question carries 4 marks.

23. (a) Find the plane tangent to the surface $z = x \cos y - ye^x$ at (0, 0, 0).

    (b) Find an equation for the tangent to the ellipse $\frac{x^2}{4} + y^2 = 2$ at (–2, 1).

24. Show that $2x\,dx + 2y\,dy + 2z\,dz$ is exact and evaluate $\int_{(0,0,0)}^{(2,3,-6)} 2x\,dx + 2y\,dy + 2z\,dz$.

25. Find a potential f for the field $\mathbf{F} = (y \sin z)\, i + (x \sin z)\, j + (xy \cos z)\, k$.

26. Use Green's theorem to find the area of the region enclosed by $r(t) = (a \cos t)\, i + (a \sin t)\, j$, $0 \le t \le 2\pi$.

27. Find the area of the cap cut from the hemisphere $x^2 + y^2 + z^2 = 2, z \ge 0$ by the cylinder $x^2 + y^2 = 1$.

28. If a, b, c, are the roots of the equation $x^3 + px^2 + qx + r = 0$. Find the equation whose roots are $bc - a^2, ca - b^2, ab - c^2$.

29. Solve the equation $x^5 - 5x^4 + 9x^3 - 9x^2 + 5x - 1 = 0$.

30. State Descarte's rule of signs and apply if to prove that the equation $x^3 + 2x + 3 = 0$ and one negative and two imaginary roots.

31. Use bisection method to obtain a root correct to three decimal places the equation $x^3 - 5x + 3 = 0$.

(6 × 4 = 24)

Part D

Answer any **two** *questions.*
Each question carries 15 *marks.*

32. (a) Find the flux of the field $\mathbf{F}(x, y, z) = -i + 2j + 3k$ across the rectangular surface $z = 0, 0 \le x \le 2, 0 \le y \le 3$, and in the direction k.

(b) Integrate $g(x, y, z) = y + z$ over the surface of the wedge in the first octant bounded by the co-ordinate planes and the planes $x = 2$ and $y + z = 1$.

33. (a) Use the surface integral in stokes's theorem to calculate the circulation of $\mathbf{F} = x^2\, i + 2xj + z^2 k$ around the ellipse $4x^2 + y^2 = 4$ in the xy plane, counter clockwise.

(b) Verify the circulation form of Green's theorem on the annular ring R :

$$h^2 \le x^2 + y^2 \le 1,\ 0 < h < 1 \text{ if } \mathrm{M} = \frac{-y}{x^2 + y^2}, \mathrm{N} = \frac{x}{x^2 + y^2}.$$

34. (a) Solve by Cardan's method :

$$x^3 - 12x - 65 = 0.$$

(b) Solve by Ferrari's method :

$$x^4 + 2x^3 - 7x^2 - 8x + 12 = 0.$$

(c) Solve $x^4 + 4x^3 - 5x^2 - 8x + 6 = 0$ given that sum of two roots is zero.

35. (a) Use iteration method to find correct to four significant figures, a real root of $\sin x = 10(x - 1)$.

(b) Find a real root of $x = e^{-x}$ by Newton Raphson method.

(2 × 15 = 30)

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