E 7194 (Pages : 3) Reg. No.....

Name.....

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, MARCH 2014

Fourth Semester

Core Course—VECTOR CALCULUS, THEORY OF EQUATIONS AND NUMERICAL METHODS

(Common for Mathematics Model I, II and B.Sc. Computer Applications)

Time : Three Hours Maximum Weight : 25

Part A (Objective Type)

Answer **all** *questions.*

A bunch of 4 *questions has weight* 1.

I. 1. Find the parametric equation of the line through (1, 1, 1) parallel to z-axis.

2. Find the plane through (0, 0, 1), (2, 0, 0) and (0, 3, 0).

3. Find the unit tangent vector to the curve $r(t) = 6t^3 i - 2t^3 j - 3t^3 k, \quad 1 \le t \le 2$.

4. Find the direction in which $f(x, y) = \frac{x^2}{2} + \frac{y^2}{2}$ increases most rapidly.

II. 5. Find the gradient of the function $f(x, y) = y - x$ at $(2, 1)$.

6. Find the gradient field of $f(x, y, z) = xyz$.

7. Find the divergence of $F(x, y) = \left(x^2 - y\right)i + \left(xy - y^2\right)j$.

8. Write the equation of the parametric formula for the area of a smooth surface.

III. 9. State Stoke's theorem.

10. If $F = \left(x^2 - y\right)i + 4zj + x^2 k$. Find curl F.

11. If α, β, γ are the roots of $f(x) = 0$, write the equation whose roots are $\frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\gamma}$.

12. Form an equation whose roots are the negatives of the roots of $x^3 - 6x^2 + 8x - 9 = 0$.

Turn over

IV. 13. If α, β, γ are the roots of $ax^3 + 3bx^2 + 3cx + d = 0$, then what are the roots of $x^3 + 3Hx + G = 0$ where $H = ac - b^2$, $G = a^2d - 3abc + 2b^3$.

14. Transform the equation $x^3 - \frac{5}{2}x^2 - \frac{7}{18}x + \frac{1}{108} = 0$ into an equation with integral co-efficients and leading co-efficient unity.

15. Find two numbers a and b such that a real root of $x^3 - 2x - 5 = 0$ lies between them.

16. Write the Taylor series expansion of $f(x)$ about $x = a$.

(4 × 1 = 4)

Part B (Short Answer)

Answer any **five** *questions.*
Each question has weight 1.

17. Find the distance from S(1, 1, 3) to the plane $3x + 2y + 6z = 6$.

18. Show that curvature of a circle of radius a is $\frac{1}{a}$.

19. Integrate $f(x, y) = x + y$ over the curve $x^2 + y^2 = 4$ in the first quadrant from (2, 0) to (0, 2).

20. Find the work done by $F = xyi + yj - y2\bar{k}$ over a curve $r(t) = ti + t^2j + t\bar{k}$ $0 \le t \le 1$ in the direction of increasing f.

21. Evaluate integral $\int_C xy\,dy - y^2\,dx$ where C is the square cut from the first quadrant by the lines $x = 1$ and $y = 1$.

22. Solve the equation $x^4 - 12x^3 + 49x^2 - 78x + 40 = 0$ by removing its second term.

23. Solve the equation $4x^4 - 20x^3 + 33x^2 - 20x + 4 = 0$.

24. Obtain a roof correct to 3 decimal places using bisection method for $x^3 - x - 4 = 0$.

(5 × 1 = 5)

Part C (Short Essay)

Answer any **four** *questions.*
Each question has weight 2.

25. Find an equation for the cylinder made by the lines parallel to the z-axis that pass through the parabola $y = x^2,\ z = 0$.

26. Show that $ydx + xdy + 4dz$ is exact and evaluate the integral $\int_{(1,1,1)}^{(2,3,-1)} ydx + xdy + 4dz$.

27. Find the area of the band cut from the paraboloid $x^2 + y^2 - z = 0$ by the planes $z = 2$ and $z = 6$.

28. Solve by Cardan's method $x^3 - 15x - 126 = 0$.

29. If α, β, γ are the roots of $x^3 + Px^2 - 9x + r = 0$ form the equation whose roots are $\frac{\beta\gamma}{\alpha}, \frac{\gamma\alpha}{\beta}, \frac{\alpha\beta}{\gamma}$.

30. Find the roots of the equation $2x = \cos x + 3$ correct to 3 decimal places by iteration method.

$(4 \times 2 = 8)$

Part D

Answer any **two** *questions.*
Each question has weight 4.

31. Use Stokes theorem to evaluate $\int_C F.dr$ if $F = xzi + xyj + 3xzk$ and C is the boundary of the portion of the plane $2x + y + z = 2$ in the first octant, traversed counter clockwise.

32. Solve the equation $x^5 - 5x^4 + 9x^3 - 9x^2 + 5x - 1 = 0$.

33. Use Newton-Raphson method to obtain a root correct to 3 decimal places for the equation $4(x - \sin x) = 1$.

$(2 \times 4 = 8)$