E 2793 (Pages : 3) Reg. No.................................

Name...................................

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, MARCH/APRIL 2012

Fourth Semester

Core Course

VECTOR CALCULUS, THEORY OF EQUATIONS AND NUMERICAL METHODS

[Common for (1) Model I — Mathematics
(2) Model II — Mathematics and
(3) B.Sc. Computer Applications]

Time : Three Hours Maximum Weight : 25

Part A (Objective Type Questions)

Answer **all** *questions.*
Each bunch of 4 questions has weight 1.

1. Write a parametric equation of the line passing through $P(3,-4,1)$ and parallel to the vector $i+j+k$.
2. Find an equation for the plane through $P_0(0,2,-1)$ normal to $n=3i-2j-k$.
3. For what values of c, the elliptical paraboloid $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{z}{c}$ lies above the xy - plane.
4. Find $\lim_{t\to\left(\frac{\pi}{4}\right)} r(t)$, where $r(t)=(\cos t)\,i+(\sin t)\,j+tk$.
5. Find the unit tangent vector of the curve $r(t)=t^2\,i+(2\cos t)\,j+(2\sin t)\,k$.
6. Find the gradient field of $g(x,y,z)=xy+yz+xz$.
7. Give an example of a conservative field.
8. Find the k-component of the curl of the vector field $F(x,y)=\left(x^2-y\right)i+\left(xy-y^2\right)j$.
9. Find a parametrization of the paraboloid $z=x^2+y^2,\ z\le 4$.
10. What is $\nabla\times\nabla f$?

Turn over

11. If α, β, γ are the roots of $2x^3 + 3x^2 - x - 1 = 0$, find the equation whose roots are $\alpha - 1, \beta - 1, \gamma - 1$.

12. If $\alpha, \beta, \gamma, ...$ are the roots of $f(x) = 0$, find the equation whose roots are $\frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\gamma}, ...$

13. Give an example of a reciprocal equation of degree 5.

14. Write the standard form of a cubic equation.

15. Find two numbers a and b such that a real root of $f(x) = x^3 - x - 4 = 0$ lies between a and b.

16. In the method of false position, what we are replacing the part of the curve between $(x_0, f(x_0))$ and $(x_1, f(x_1))$, where a real root of $f(x) = 0$ lies between x_0 and x_1 ?

(4 × 1 = 4)

Part B (Short Answer Type Questions)

Answer any five questions.
Each question has weight 1.

17. Show that if $u = u_1 i + u_2 j + u_3 k$ is a unit vector, then the arc length parameter along the line $r(t) = (x_0 + tu_1) i + (y_0 + tu_2) j + (z_0 + tu_3) k$ from the point $P(x_0, y_0, z_0)$ where $t = 0$ is t itself.

18. Find the derivative of $f(x, y) = 2xy - 3y^2$ at $P_0(5, 5)$ in the direction of $A = 4i + 3j$.

19. Find the circulation of the field $F = (x - y) i + xj$ around the circle $r(t) = (\cos t) i + (\sin t) j$; $0 \le t \le 2\pi$.

20. Find the work done by $F = (x^2 + y) i + (y^2 + x) j + z e^z k$ over the line segment $x = 1$, $y = 0, 0 \le 2 \le 1$.

21. Using Green's theorem, find the outward flux of the field $F(x, y) i + (y - x) j$ across the square bounded by $x = 0, x = 1, y = 0, y = 1$.

22. Solve the equation $x^4 + 6x^3 - 5x^2 + 6x + 1 = 0$.

23. Solve $6x^3 - 11x^2 - 3x + 2 = 0$, given that the roots are in harmonic progression.

24. Write the condition for the sequence of approximations to a real root of an equation $f(x) = 0$ converges to the required root in the method of iteration.

(5 × 1 = 5)

Part C (Short Essay Type Questions)

Answer any **four** *questions.*
Each question has weight 2.

25. Find the curvature for the helix $r(t) = (a \cos t) i + (a \sin t) j + bt\, k ; a, b \geq 0, a^2 + b^2 \neq 0$.

26. Verify both forms of Green's theorem for the field $F(x, y) = (x - y) i + xj$ and the region R bounded by the unit circle $C : r(t) = (\cos t) i + (\sin t) j ; 0 \leq t \leq 2\pi$.

27. Integrate $g(x, y, z) = xyz$ over the surface of the rectangular solid cut from the first octant by the planes $x = a, y = b$ and $z = c$.

28. If α, β, γ are the roots of $x^3 + qx + r = 0$, prove that the equation whose roots are $\frac{\beta}{\gamma} + \frac{\gamma}{\beta}, \frac{\gamma}{\alpha} + \frac{\alpha}{\gamma}, \frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ is $r^2 (z+1)^3 + q^3 (z+1) + q^3 = 0$.

29. Find the five places of decimals the real root of $x^3 + 6x^2 + 27x - 26 = 0$.

30. Find a real root of the equation $\sin x = 10(x - 1)$ using iterative method.

$(4 \times 2 = 8)$

Part D (Essay Type Questions)

Answer any **two** *questions.*
Each question has weight 4.

31. Find the area of the surface cut from the bottom of the paraboloid $x^2 + y^2 - z = 0$ by the plane $z = 4$.

32. Prove that every polynomial equation of the n^{th} degree has n and only n roots.

33. Find a real root of $x^3 - 5x + 3 = 0$ using Newton-Raphson method.

$(2 \times 4 = 8)$